THE GROVND OF ARTES

Teachyng the worke and practise of

Arithmetike.

Bothe in whole numbres and Fractions,

After a more easyer and exacter sorte, than any lyke hath hytherto been sette forth: with diuers new additions, as by the table doeth partely appeere.

Made by M. ROBERT RECORDE Doctor of Physike.

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Cum Sereniß. Regis priuilegio.

The Bookes Verdicte.

To please or displease, sure I am,
But not of one sorte to euery man.
To please the beste sorte woulde I fayne,
The frowarde to displease I am certayne.
Yet wyshe I wyll, though not with hope,
All mouthes and eares to please or stoppe.



TO THE MOSTE CHRISTIAN PRINCE EDWARD THE syxte, by the grace of God, kynge of Englande, Fraunce, and Irelande, defendour of the fayth, and of the churches of England, and Ireland in earth, the supreme heade.



The excellency of mans nature, beeyng suche, as it is by goddes dyuine fauoure, (moste myghtye prince) not onely created in hyghnesse of degree, farre aboue all other corporalle thynges, but by perfection of reason and searche of wytte muche approchyng toward the ymage of god, as not only the holy scriptures doo testifye, but also those naturall philosophers, whiche exactely dyd consyder the nature of man, & namely the farre reache and infinite compasse of the woorkes of the mynde, were inforced to confesse, that man scarsely was able to knowe hym selfe. And if he wold duly ponder the nature of hym self, he shuld fynde it so straunge, that it mygbte

seme vnto hym a very myracle, and thereof
wrange that sayeng: Magnum miraculum
est homo, maximum miraculum sapiens ho
mo. For vndoubtedly as man is one of the
greatest myracles, that euer god wrought,
so a wyse man is playnely the greatest. And
therfore was it, that some dyd accompt the
head of a man to be the greatest myracle in
the worlde, bycause not onely of the strange
workmanshyp that is in it, but muche more
of the efficacie of reason, wit, memory, ima-
gination, and such other powers and works
of the mynde, whiche can more easyly con-
ceue any thyng in a maner, than vnderstand
it selfe, and amongest al the creatures of god
it findeth none more difficult to be perceiued
then these same powers of it selfe, whereby it
doth conceiue & iudge, as it may be well con-
iectured by the diuersitie of opinyons that
the wysest philosophers did vtter, touchyng
the spirite of man & the substãce of it: wherof
at this present I intend to make no reher-
sal, but whoso listeth to read therof, may find
it largely set foorth not only in Aristotle his
bokes De anima, but also in Galen his boke
called Historia philosophica, & again in Plu
tarche his worke De philosophorũ placitis,
whose words ar also repeted of Eusebius in
the 15 boke, τῆσ εὐαγγελικῆς προπαρα-
σκευῆς, vnto whõ I remit them that haue de-
sire to vnderstande the intricate difficulter of

kno-

knowyng our awne selues, as touchyng our best part, and that parte whereby we deserue to beare the name of men. This matter semed so obscure & difficile in knowledge, that Galen, who for his excellent wisdom & iugement in naturall workes, is called of many men a myracle in nature, yet in setthyng the nature & substance of the spirite of man, he not only confesseth him self ignorãt, but coũteth plain temeritie to attempt to finde it, so farre aboue the hope of mans knowledge is that part whereby man doth know and iudge of thinges. And although the ignorant sort (which hate all thinges that they know not) do litle esteme the profoundnes of mans spirite & of reason, the chief power and faculties of it, yet as there is a kynd of feare and obedience of all vnreasonable beastes vnto mã, by the workyng power of god, so is there in those small reasoned persons a certayn kynd of reuerence toward wysedome and reason, whiche they do shew oftentymes. and by power of persuasion are inforced to obey reason, wyll they nyll they. And hereby came it to passe, that the rudenesse of the fyrst age of man was brought vnto somme more ciuile trade as it is well declared by Cicero in the begynnynge of his fyrst boke De inuentione Rhetorica, where he sayth thus.

Nam fuit quodd ã tempus quũ in agris homines passim bestiarum more uagabantur, & sibi

victu ferino vitam propagabant, nec ratione animi quicq̃, sed plæraque viribus corporum administrabant. Non dum diuinæ religionis, non humani officij ratio colebatur. Nemo legitimas viderat nuptias, non certos quicq̃ inspexerat liberos, non ius æquabile quid vtilitatis haberet, acceperat: ita propter errorem atque insciti am cæca ac temeraria dominatrix animi cupiditas ad se explendam viribus corporis abutebatur, pernicioſiſsimis ſatellitibus. Quo tempore quidã, magnus videlicet vir & ſapiens, cognouit quæ materia eſſet, & quanta ad maximas res oportunitas in animis meſſet hominum, ſignis eam poſſet elicere, & præcipiendo meliorem reddere. Qui diſperſos homines in agris, & in tectis ſyluestribus abditos, ratione quadam compulit in vnum locum, & congregauit: & eos in vnam quanque rem inducens vtilem atque honeſtã primo propter inſolentiã reclamantes, deinde propter rationẽ atque orationẽ ſtudioſius audientes, ex feris & immanibus mites reddidit, & manſuetos. This long repetition of Tullies wordes wyll ſeme tedious to them whiche loueth but lytle, and careth muche leſſe for the knowlege of reaſon, but vnto youre maieſtie (I dare ſay) it is a delectable remẽbrance) and vnto me it ſemed ſo pleaſaunt, that I could ſcarſe ſtay my penne from wryting all, that myn eies did ſo gredyly read.

This ſentence of Cicero am I lothe to trãſlate into engliſh, partly for that vnto youre maieſtie it nedeth no tranſlation, but eſpecially

ally knowyng howe farre the grace of Tullies eloquence doth excell any englyſh mans tongue, and muche more exceedeth the baſeneſſe of my barbarous ſtyle, yet for the fruit of the ſentence, I had rather vnto my mere englyſhe countrey men vtter the rudenes of my tranſlation, then to defraude them the benefite of ſo good a leſſon, truſtyng, they wyll ſo learne to loue reaſon, that they wyll alſo gladly and gredily imbrace all good ſciences that may helpe to the iuſt furniture of the ſame, when they conſider that informed reaſon was the onely inſtrument, or at leaſte the chiefe meane to bryng men vnto ciuile regiment, from barbarous maners & beaſtly cõditions. For the tyme was (ſayth Tullie) that men wandered abrode in the fieldes vp and downe, lyke beaſtes, and vſed no better order in feedyng then they, ſo that by reaſons rule, they wrought nothyng, but moſte of theyr dooynge dyd they atcheue by force of ſtrengthe. At this tyme, there was no iuſte regarde of religion, towarde God, nor of duetie towarde man. No man hadde ſeene tyght vſe of maryage, nother dyd any man knowe theyr owne chyldren from other, nor no man had felte the commoditie of iuſte lawes, ſo that thorough errour and ignorannce, wylfull luſte as a blynde and heady ruler abuſed bodyly ſtrength as a moſt mortall miniſter, for the ſatisfying of his deſyre.

At that tyme was there one, whyche not onely in power, but also in wysedome was greate, And be conlydered how that in the myndes of men was bothe apte instrumentes, and great occasion to the greate accomplishement of moste weighty affayres, yf a man coulde applye theym to vse, and by teachyng of rules, frame them to better trade. This man with persuasyon of reason, gathered into one plate the people that were wanderynge aboute the fieldes, and laye lurkyng in wylde corages and woddes. And bryngyng theim into one common societie, hyd trade theim to all suche thynges, as eyther were profitable or honest, although not without repinyng at the fyrst, by reason that they had not ben so accustomed before, yet at length thorough reason and persuasion of wordes they obeyed hym more diligentely, and so of a wyld and cruell people, he made them curteous and gentyll.

Thus hath Tullie set forthe the efficacie of reason and persuasion, howe it was able to conuert wylde people to a myldues, and to chaunge their furious cruelnes into gentill curtesy, were it not now a great reproche in this oure tyme (whenne knowledge reigneth so large) that men shoulde shewe them selfe lesse obsequious to reason? ouelesse it may be thought, that nowe euery man hauyng sufficient knowledge of hym selfe, ne-

deth

deth not to harken to the perſuation of o-
thers. In dede he that thynketh hym ſelfe
wiſe, wyl not eſteme the reaſon of any other,
be he neuer ſo wiſe, ſo that of ſuch one it may
wel be ſaid, He that thynketh himſelf wyſer
then he is, may iuſtly bee counted a double
foole: wherfore ſuch men are not to be permit
ted in open audience to talke, but muſte bee
put to ſilence, & made to giue eare to reaſon,
whiche reaſon conſiſteth not in a multitude
of wordes, heaped rathely togither, and ap-
plied for one purpoſe, but reaſon is therpre-
ſyng of a ruſte matter, with witty perſuaſi-
ons, furnyſhed with lerned knowledge: ſuch
knowlege had Moyſes, beyng expert in all
lernyng of the Egyptians, as the ſcriptures
declare, & therfore was hable to perſuade the
ſtubborn people of the Iewes, although not
without great peyn: ſuch knowledge & ſuche
reſons did Numa ſhew, which was the firſt
lawmaker of al the weſt part of Europe: like
reaſon and wyſedome dyd Lamoxis vſe a-
mongeſt the Gothes: Lycurgus vnto the
Lacedemonians: Zaleucus to the Locrias:
Solon to the Athenienſes, and Dunwallo
Molmitius 2000 yeres paſte, amongeſt the
olde Britayns of this realme: & therby came
it to paſſe, y their lawes continued long tyll
more perfect reaſon altered many of them, &
wilfull power oppreſſed moſte of them. At
the beginnyng whē theſe wiſe mē perceiued

howe harde it was to bryng the rude people to vnderstand reason, they iudged the beste meanes to attayne this honest purpose, to depende of learnyng in euery kynde, for by learnyng, as Ouide sayth: pectora molle-scunt, asperitasque fugit. Stoute stomaches doo wexe mylde, and sharpe fiercenes is exiled. Therfore as Berosus doth testify, Sarron, that was the thyrde kyng ouer all this west parte of Europe, for to bryng the people from beastly rage to manly reason, dyd erecte schooles of liberall artes, which toke so good succesle, that his name continued in that sorte famous aboue 2000 yeares after, for Diodorus Siculus, whiche was in the tyme of Julius Cesar, maketh mention of the lerned men, of the Celtes, and nameth theym Sarronides, that is to say, Sarron his scholars and folowers.

Among these artes that then were taught some dyd informe the tonge, and make men hable bothe to vtter aptly their mynde, and also to persuade, as Grammar, Logike, and Rhetorike, although not so curiously as in this tyme, som other dyd appertayne to the iuste order of partition of landes, the trewe vsyng of weyghtes, measures, and reckynynges, in all sortes of bargaynes, and for order of buildyng and sundry other bles. those wer Arithmetike and Geometry. Agayn to encourage men to the honour of God, they

taught

taught Aſtronomy, whereby the wonderfull worckes of god were ſo manyfeſtly ſet forth, that no mans tongue nor penne can in lyke ſort expreſſe his infinite power, his vnſpeakable wyſedome, and his exceedynge goodneſſe towarde man, whereby he dothe bountifully prouide for man all neceſſaries, not onely to lyue, but alſo to lyue pleaſantly. And ſo was their confidence in gods prouidence ſtrongly ſtayed, knowyng his goodnes to be ſuche that he woulde helpe man as he coulde, and his power to be ſo great, that he coulde doo what he woulde, and thyrdly his wyſedome to be ſo pure, that he would doo nothynge, but that that was beſte. Beſyde theſe ſciences they taught alſo Muſike, whiche moſte commonly they dydde apply partely to religious ſeruices, to drawe men to delyte therin, and partely to ſonges made of the maners of men, in prayſe of vertue, and diſcommendation of vice, whereby it came to paſſe, that no man would diſpleaſe them, nor doo any thyng euyll, that myghte come to theyr hearynge, for theyr ſonges dyd make euyll men more abhorred in that tyme, then any excommunication doothe in this tyme. The poſteritie of theſe Muſitians contynue yet doothe in Wales and Ireland, called Bardes vnto this daye, by the ancient name of Bardus, theyr fyrſt founder.

And as theſe ſciences dyd increaſe, ſo dyd

vertue

vertue interese therby. Agayn as these sciẽces did decay, so vertue lost her estimation, & consequently was lytle in vse. wherof to make a full declaration were a thynge mete for a Prince to heare, but it wouldde requyre a particular treatise. wherefore at this present I counte it sufficient lyghtly to haue touched this matter in generall woordes, and to say no more of the particularities therof, but only touchyng one of those sciences, that is Arithmetike, by whiche not onely iuste partition of lãdes was made, but also touchyng byenge and sellynge all assises, weyghtes, and measures were diuised, and all reckynynges and accomptes deuysed, yea by proportion of it, were the true orders of Justice limytted, as Aristotle in his Ethikes doth declare, and the degrees of estates in the commune wealthe established. Although that proportion bee called Geometricall, and not Arithmeticall, yet doothe that proportion appertayne to the arte of Arithmetike, and in Arithmetike is taught the progression of suche proportions, and all thynges therto belonging. Wherfore I may well say, that seing Arithmetike is so many wais nedefull vnto the fyrst plantyng of a comon wealthe, it must nedes be as muche required to the preseruation of it also, for by the same meanes is any common wealthe continued, by whyche it was erected and established.

And

And if I shal in smal matters in apperance, but in deede very weyghty, put one example or two. what shall wee saye for the statutes of this Realme, whyche bee the onely stay of good order in maner nowe? As touchyng the measuryng of ground by lengthe and bredthe, there is a good and an ancient statute made by art of Arithmetik, and now it shalbe to litle vse, if by the same arte it bee not practised and tried: for the assise of bread and drynke, the two most common and most nedessary thynges for sustentation of man, there was a goodly ordynaunce in the lawe made, whyche by ignoraunce hathe so growen out of knowledge and vse, that fewe men doo vnderstande it, and therfore the statute bookes wonderfully corrupted, and the commons cruelly oppressed, not withstandynge some men haue written, that it is to doubtfull a matter to execute those assises by those statutes, by reason they depend of the standerd of the corne, whyche is muche chaunged frome the state of that tyme whanne those statutes were made. Thus shall euery man reade (that lysteth) in the Abridgemente of statutes, in the tytle of weightes and mesures, in the seuenth number of the Englyshe booke, where he should haue translated a good ordinance, whych is sette foorth in the French boke: but no mervayle yf the Abridgement dooth omytte it,

seyng

seyng the great bake of statutes doth dimpe the same statute, as it hath doon dyuers other very good lawes. And this is the fruit of ignorance, to reiecte and condemne all that it vnderstandeth not, although they vse some clokes for it, but suche clokes as beyng allowed, might serue to repel all good lawes, whiche God forbyd.

Agayn there is an auncient order for assise of fyre wood, and coales, whiche was renewed not many yeres paste, and nowe howe Auarice and Ignorance doth ranuas that statute, it is to pitifull to talke of, and more myserable to feele.

Farthermore for the statute of coynage, and the standerde therof, if the people vnderstoode rightely the statute, they shuld not nor wolde not (as they often do) gather an excuse for their foly therby, but as I sayde, these statutes by wysedom and good knowlege of Arithmetike were made, and by the same must they be continued. And let ignorance no more meddle with the vse of them, then it did with the makyng of them. Oh in howe myserable case is that realme, where the ministers and interpreters of the lawes are destitute of all good sciences, whiche be the keyes of the lawes? Howe can they eyther make good lawes, or maynteyne them, that lacke that true knowledge, whereby to iudge them. And happy may that realme be

accomp-

accompted, where the Prince hym selfe is studious of learnyng, and delyreth to vnderstand equitie in al lawes. Therefore most happy are we the louyng subiectes of your maiestie, whiche may see in your highnes not only suche towardnes, but also suche knowledge of hymers artes, as seldome hath bene sene in any prince of such yeres, whereby we are inforced to conceaue this hope: Certainly, that he which in those yeres seketh knowledge when knowledge is leall esteemed, and at suche an age can discerne theim to be enemyes bothe to his royall person, and to his realmes, whiche labour to withdrawe hym frome knowledge to excellent pastyme, and from reasonable study to ydle or noysom pleasures, he muste nedes whan he commeth to more mature yeares bee a moste prudente prince, a moste iuste gouernour, and a right iudge, not only of his subiectes commonly, but also of the mynisters of his lawes, yea and of the lawes them selfe. And to be able to conceaue the true equitie and exacte vnderstandynge of all his lawes and statutes, to the comforte of his good subiectes, and the confusion and reproche of theim whiche labour to obscure or peruerte the equitie of the same lawes and statutes. How som of those statutes may bee applied to vse, as well in this our tyme, as in any other tyme, I haue peculiarly declared in this boke, and some

other

other I haue omitted for iuſt conſiderations
tyll I may offer them fyrſt vnto your Maie-
ſtie, to way them, as to your byghnes ſhall
ſeeme good: for many thynges in them are
not to be publiſhed without your hyghneſſe
knowledge and approbation, namely be-
cauſe in them is declared all the rates of al-
loies for all ſtanderdes frome one vnce vp-
ward, with other myſteries of mynte mat-
ters, and alſo moſte part of the varieties of
coynes that haue ben currant in this youre
Maieſties realme, by the ſpace almoſt of vi.
hundred yeares laſt paſt, and many of them
that were currant in the tyme that the Ro-
maines ruled here.

All whiche with the ancient deſcription of
England and Irelande, and my ſimple cen-
ſure of the ſame, I haue almoſte completed,
to be exhibited to your byghnes. In the mean
ſeaſon moſt humbly beſeking your Maieſty
to accept this ſimple treatiſe, not worthy to
bee preſented to ſo hygh a Prince, but that
my lowely requeſt to your Maieſtie is, that
this amongeſt other of my bokes, may paſſe
vnder the protection of youre hyghneſſe,
whom I beſeke God moſt earneſtly, & daily,
accordyng to my duetie, to aduance in all ho
nour and princely regalitie, and to increaſe
in al knowlege, iuſtice, & godly policy. Amen

Your Maieſties moſt obedient ſub-
iect & ſeruant, Robert Recorde.

TO THE LOVYNG READERS.



Dfte ofttymes haue I lamented with my self the infortunate condition of England, seyng so many great clerkes to aryse in sundry other partes of the worlde, and so few to apere in this our nation: where as for pregnancy of naturall witte (I thinke) fewe nations doo excell englyshemen. But I can not impute the cause to any other thynge, than to the contempt or mysregarde of learnynge. For as Englyshmen are inferiour to no men in mother witte, so they passe all men in vaine pleasures, to whiche they may atteyne without great peyne or labour, and are as slacke to any newer so great cõmoditie, if there hange of it any peynfull study or trauelsome labor: Howbeit, yet all men are not of that sorte, though the moste parte bee, the more pitie it is: but of them that are so glad not onely with paynfull study and studyous payne to attayne lernyng, but also with as great study and payne to cõmunicate their learnyng

to other, & make all Englande (yf it mought bee) partakers of the same, the moste parte are such, that vnneth they cã suppoꝛte their owne necessarye charges, so that they are not able to beare anye charges in doyng of that good, that els they desire to doe. But a greater cause of lamẽtacion is this, that when learned men haue taken paynes to doe thynges for the ayde of the vnlearned, scarse they shal bee allowed for their welldoyng, but derided and scorned & so vtterly discouraged to take in hand any like enterpꝛyse agayn. So that if any be found (as there are some) that dooth fauour learnyng & learned mattes, and can be cõtent to further knowledge, yea onely with theyr woord, such persons, though they bee pore, yet shall they encourage learned menne to enterpꝛyse some thynges, at the leaste, that Englãde maye reioyse of. And I haue good hope, that Englãde wyll (after she hath takẽ some sure taste of learnyng) not onely bryng foorthe more fauourers of it, but also suche learned men, that she shall be able to cõpare wyth anye Realme in the woꝛld. But in the meane season, where so fewe regarders of learnyng are, howe greatly they are to bee esteemed that dooe fauour and further it, my penne wyll not suffice at full to declare. Therefoꝛe gentle reader, where as I dooe vpon moste iuste occasion iudge, yea and know

knowe assuredly, that ther be sum mẽ in this reame whiche bothe loue, and also muche delyre to further good learnynge, and yet am I not well able to write ther condigne prayse for the same, I thinke it better with silence to ouerpasse it, than other to saye to litle of it, or to prouoke agaynste them the malice of suche other, whiche doo nothynge them selfe that is prayse worthy, and therfore can not abyde to heare the prayse of anyother mannes good dede. And consyderyng thyr greate fauour vnto learnyng thoughe I my selfe bee not worthye to bee reckened in the numbre of greate learned men, yet am I bolde to put my selfe in prealle wyth suche abilitee as God hathe lente me, thoughe not wyth so great cunnynge as many men, yet wyth as great affection as any man to helpe my countrey men, and wyll not ceasse dayly (as muche as my smale abilitee wyll suffer me) to endyse some suche thynge that shall bee to the instruction, thoughe not of learned men, yet at the least of the vulgar sorte, whose argument alwayes shall be suche, that it shall delyte all learned wyttes, thoughe they doo not learne any great thynges out of it. But to speake of thys present booke of Arithmetike, I dare not nor wyl not set it foorthe wyth anye woordes, but remytte it to the iudgements of all gentyll

readers, and namely such as loue good learnyng, beseechyng them so to esteeme it, as it doothe seme worthy. And so, other to accept the thyng for it selfe, other at the least to allowe my good endeuour. But I perceaue I nede not vse any persuasions vnto theym, whose gentyll nature and fauourable mynd is ready to receyue thankefullly, and interprete to the best all suche enterprises attempted for so good an ende, though the thynge doo not alwaies satisfie mens expectation. This considered, dyd bolden me to publyshe abroade this lyttle booke of the arte of numbryng, which if you shall receyue fauourably, you shall encourage me to gratifye you hereafter with some greater thyng. And as I iudge some men of so louynge a mynd to theyr native countrey, that they wold muche reioyce to see it to prosper in good learnyng and wytye artes, so I hope well of all the reste of Englyshemen, that they wyll be not vnmyndefull of his dewe prayse, by whose meanes they are healped and furthered in any thynge. Neyther oughte they to esteme this thynge of so lyttell valewe, as manny men of lyttell discretion oftentymes doo: for who so setteth small prycе by the wytye deuise and knowledge of numbryng, he lyttell considereth it to be the chiefe poynt (in maner) whereby men differe from all brute beastes: for as in all other thynges (almoste)

beastes

beastes are partetakers with vs, so in numbryng we differ cleane from them, and in maner peculiarly, sythe that in many thynges they excell vs agayne.

The Foxe in crafty witte excedeth moste men,
A Dogge in smellyng hath no man his peere.
To foresyght of wether, yf you loke then,
Many beastes excell man, this is cleere.
The wittinesse of Elephantes doth letters attayn
But what cunnyng doth there in the Bee remayne?
The emmete foreseyng the hardnesse of wynter,
Prouideth vytailes in tyme of sommer.
The nyghtyngale, the lynet, the thrushe, the larke
In Musicall harmonie paße many a clarke.
The hedgehogge of astronomy semeth to know,
And stoppeth his caue wher the wind wil blowe
The spyder in weauyng, suche arte doth showe,
No man can hym mende, nor folow, I trowe.
When a house wyll fall, the myse ryght quicke,
Flee thens before, can man doo the lyke?

Many thynges els of the wyttynesse of beastes and bydes myght I here say (saue that an other tyme of them I entende to wryte) wherin they excell in maner all men, as it is daiely seene: but in nūbre was there neuer beast founde so counnyng, that coulde knowe or discerne one thyng from many, as

by dayely experience you maye well consider, whan a bytche hath many whelpes, or a henne many chyckens: and lyke wayes of other, what so euer they be, take from them all they? yonge, sauyng onely one, and you shall perceiue plainly, that they misse none, though they wyll resyst you in takyng them awaye, and wyll seeke them agayne, yf they may knowe where they be: but elles they wyll neuer mysse theim truely: but take awaye that one that is lefte, and then wyll they cry and complayn: and restore to theim that one, then are they pleased agayne: so that of numbre this may I iustly say, It is the onely thynge (all moste) that separateth manne from beastes. He therefore that shall contempne numbre, he declareth hym selfe as brutyshe as a beaste, and vnworthy to be counted in the felowshyp of menne. But I truste there is no manne so foule ouerseene, though many ryght smally doo it regarde. Therfore wyll I nowe stay to write against suche, and retourne agayn to this my boke, whyche I haue written in the fourme of a Dialogue, because I iudge that to be the easyest waie of instruction, whan the scholar maie aske euery doubt orderly, and the maister maie answere to his question playnely. Howe beit, I thynke not the contrary, but as it is easyer to blame an other mannes woorke than to make the lyke, so there wyll be

bee some that wyll fynde faulte, because I wryte in a Dialogue: but as I coniecture those shall be suche, as doo not, can not, either wyll not perceyue the reason of ryghte reachyng, and therfore are vnmete to be answered vnto, for suche men with no reason wyll bee satisfied. And if any man obiecte that other bookes haue been written of Arithmetike all ready so sufficientely, that I needed not nowe to put penne to the booke, except I wyll condemne other mennes wrytynges: to them I answere, That as I condemne no mans diligencie, so I know, that no one man can satisfie euery manne: and therfore lyke as many dooth esteme greatly other bookes, so I doubt not, but some will lyke this my booke aboue any other Englyshe Arithmetike hytherto written, and namely suche as shall lacke instructers, for whose sake I haue so playnely set forthe the examples, as no booke (that I haue seene) hath doone hytherto: whyche thyng shalbe great ease to the rude readers. Therfore gentyl reader, though this boke can be but smal ayde to the learned sort, yet vnto the symple ignorant, (whyche nedeth most helpe) it may be a good furtherance & meane vnto knowledge. And though vnto the kyng his maiestie priuately I doo it dedicate, yet I doubte not (suche is his clemency) but & he can be cõtent, yea & much desyrous that al his louing

subiectes shall take the vse of it, and employ the same to theyr moste profit. which thyng yf I perceaue that they thankefully do, and receyue with as good wyll as it was written, then wyll I shortely vouchsafe some litle encouragement vnto further trauayle in Introductions into Geometrie and Cosmography, as I haue at tymes prompted, and as hytherto in English hathe not bene enterprysed, wherewith (I dare saye) all honeste hartes wyll bee pleased, and all studyous wyttes greatly delited. I wyll say no more, but leue euery man iudge as he shall see cause. And thus for this tyme I wyll stay my penne, committyng you all to that freewe fountayne of perefect numbre, whiche wrought the whole worlde by numbre, measure, & wyght in Trinitie, and vnitie in Trinitie, to whome bee all praise, honor, and glorye. AMEN.

The Contentes of the fyrſt Dialogue.

¶ The declaracion of the woꝛdes of Arithmetike.
Numeracion with an eaſy and large table.
Addicion.
Subtraccion.
Multiplicacion.
Diuiſion.
} with dyuers examples, and ſome newe fourmes of woꝛkyng.
Reduccion with dyuers declaracions of coynes, weyghtes, and meaſures of ſundꝛy fourmes, nowe newely added to theſe other rules.
Pꝛogreſſion bothe Arithmeticall and Geometricall, with certayne queſtions touchynge the ſame.
The golden rule, and the backer rule, with dyuers queſtions therto belongyng.
The double rule of pꝛopoꝛcion.
The rule of feloweſhyp, bothe with tyme, and without tyme.
¶ Unto all theſe is added theyr pꝛofe.

In the ſeconde Dialogue.

¶ The fyrſte two kyndes of Arithmetike wꝛought by counters.
The cõmon kyndes of caſtynge accomptes after marchantes faſhion & auditours alſo.
Numbꝛynge by the hande, newely added.

The

The contentes of the seconde part touchyng Fractions.

What a Fraction is.
Numeration in fractions.
The order of woorke in Fractions.
Multiplication.
Diuision.
Reduction
- Of diuers fractions into one denominatiō in 3 varieties.
- Of fractions of fractions.
- Of improper fractions.
- Of fractions to the smallest denomination.
- Of fractions into other partes of thynges.

Agayne of Multiplication.
Duplation.
Agayne of Diuision.
Mediation.
Addition.
Subtraction.
The Golden rule with dyuers questions.
The proofe of the Golden rule.
The Backer rule.
A question of lone.
The statute of Assise of breade and ale, recognised and applyed to this tyme, with newe tables thereto annexed.
The statute of measuryng of groundes, with a table and questions.

One

Questions of fourme, with the reason of the rule, and proues of theyr workes.
To fynde .ii. numbres in any proportion.
The rule of alligation with dyuers questions, and the proofe of theyr woorkes, and many variations of suche solutions.
The rule of Falsehod or false position, with dyuers questions, and theyr proofe.

Before the Introduction of Arithmetike these figures muste be learned.

Figures of numbre.

i.	one	xx.	twenty.
ii.	two.		&c
iii.	thre	xl.	fortye.
iiii.	foure	l.	fyftye.
v.	fiue	lx.	fyxtie.
vi.	fixe		&c.
vii.	seuen	xc.	nynetie.
viii.	eyght	C.	a hundred.
ix.	nyne	CC	two hundred
	&c.	D.	fyue hundred.
x,	ten,		&c.
xi.	aleuen	D.C.	fyxe hundred.
xv.	fyftene		&c.
	&c.	M.	thousande.
xix,	nynetene,		

Figures

Figures of money.

℔, a tee, the .xvi. ℏ, a hewe, the .viii. ℈, a farthyng, the .iiii. } parte of a penny.
ob, an halfepenny.
i d, a penny.
i, s, a shyllynge.
i, li, a pounde.

The Printer to the Reader

To take in woorthe my payne,
T'intreate it ſeemeth vayne:
No man there is I geſſe,
Wyll ſlacke ſuche thankfulnes
And where my prynt offended,
I truſte I haue amended.
And yf I haue omitted ought,
Suche faulte of mendyng neded nought.

A DIALOGVE BE-
TWENE THE MAISTER and the Scholar, teaching the art and vse of Arithmetike with the pen.

The Scholar speaketh.



Yea, such is your auctority in mine estimation, that I am content to consēt to your saying, and to receiue it as truth, though I se none other reaso that doth leade me therunto: wher as els in mine owne conceite it appeareth but vayn, to bestow any tyme priuatly in learning of that thynge, that euery chylde maye and dooth learn at al times and houres, when he doth any thyng him selfe alone, and muche more whan he talketh or reasoneth with other.

Mayster. Lo, this is the fashion and chance of al them, that seeke to defend their blinde ignorance: that when they thynke thei haue made strong reason for them selfe, thā haue they proued the quite contrary. For if numbring be so common (as thou granteſt it to be) that no man can doo any thynge alone,

and muche lesse talke or bargain with other, but he shall styll haue to doo with numbre: this proueth not numbre to be contemptible and vile, but rather ryght excellente and of hygh reputacion, syth it is the ground of al mens affayres, so that without it no tale can be tolde, no communication without it can be long continued, no bargaynyng without it can duely be ended, nor no bulynesse that man hath, iustly completed. These commodities (if there were none other) are sufficient to approue the woorthynes of numbre. But there are other vnnumerable farre passyng all these, which declare Numbre to excede all praise. Wherfore in all great woorkes, are Clerkes so much desired? Wherfore are Auditours so rychely feed? What causeth Geometrias so highly enhaced? Why ar Astronomers so greatly aduanced? because that by numbre suche thynges they do fynde, whych elles shulde farre excell mans mynde. Scholar. Verily syr if it bee so, that these men, by numbryng their cunnyng doo atteyne, at whose great woorkes most men dooe wonder, then I see well I was muche deceaued, and noumbryng is a more cunnyng thynge then I tooke it to bee. M. If numbre were so vile a thyng, as thou dydst esteme it, then nede it not to be vsed so much in mens communication. Exclude numbre and answere me to this question: How ma-

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nyppeares olde arte thou? Scho. Mum. M. Howe many daies in a weke? howe manny wekes in a yere? What landes hath thy father? How many men doth he kepe? Howe longe is it syth you came from hym to me? S. Mum. M. So that if numbre want, you answere all by Mummes: How many myle to London? Scho. A poke full of plummes. Maister. Why, thus may you see what rule Numbre beareth, and that it numbre be lacking, it maketh men dombe, so that to most questiõs, they must answer Mum. S. This is the cause syr, that I iudged it so vyle, by cause it is so cõmon in talkyng euery whyle: For plentie is no deyntie, as the cõmon sayeng is. M. No, nor store is no sore: perceiue you this? The more common that a thyng is, beyng needfully required, the better is the thyng, and the more to bee desyred. But in numbryng as some of it is lyght and playn so the moste part is difficult, and not easy to attayne. The easyer part serueth all men in commune, and the other part requireth some learnyng. Wherfore as without numbryng a man can doo almoste nothyng, so with the helpe of it, you may attayne to all thyng. S. Yea syr? Why, then it were best to learn the arte of numbryng fyrst of al other learnyng and then a man nede learne no more, yf all other come with it. Mayst. May not so, but if it be first learned, then shall a man be able

(I meane) to learne, perceaue, and attayne to other sciences, which without it, he shuld neuer gette. Scho. I perceyue by youre former wordes that Astronomie and Geometrie depend much of the help of numbryng, But that other sciences, as musike, physike, lawe and grammar, and such like, haue any helpe of Arithmetik, I perceaue not, M. I may perceaue your great clerkiynes, by the ordryng of your sciences, but I will let that passe now, because it toucheth not the matter that I entend, and I wyl shew you, how Arithmetike doth profite in all these, somewhat grossely, according to your small vnderstandyng, omittyng other reasons more substantiall.

Frst (as you reckon them) Musike hath not only great healpe of Arithmetike, but is made, and hath his perfectnes of it, for all musyke standeth by numbre and proporcion. And in physyke beside the calculation of Criticall dayes with other thynges whiche I omytte: How can any man iudge the pulce rightly that is ignorant of the proporcion of noumbres? And as for the lawe it is plaine, that the man that is ignorant of Arythmetyke is nether meete to bee a iudge neither a proctour. For how can he well vnderstand an other mans cause appertaining to distribution of goodes, or other dettes or sums of mony, if he be ignorant of Arythmetike?

rithmetik
cessary
physyk.

he lawe.

metike? This oftentymes causeth ryght to be hyndred, when the iudge eyther delyteth not to heare of a matter that he perceyueth not, eyther can not iudge it for lacke of vnderstandynge: this commeth by the ignorance of Arithmetike. Nowe as for grammar, me thinketh thou shuldest not doubt in what it neadeth numbre, sithe thou hast learned that nounes of all sortes, pronounes, verbes and participles, are distincte diuersly by numbres. besydes the varietie of nounes of numbre, and aduerbes. And if thou take away numbre frome grammar, then is all the quantitie of sylla-bles lost. And many other wayes dothe numbre helpe grammer. Gram

Wherby were all kyndes of meeters founde & made? Was it not by numbre? But howe needefull Arithmetike is to all partes of philosophy they may soone see, that readeth eyther Aristotle, Plato, or any other Philosophers writyng. For al theyr examples almoste and theire probations depende of Arithmetike. It is the sayenge of Aristotle, that he that is ignorant of Arithmetike, is mete for no science And Plato his maister wrote a lyke sentence ouer his schoolehouse doore, Let none enter in hyther (quoth he) that is ignorant of geometrie. Seeyng he wolde haue all his scholers expert in geometry, muche rather he wold the same in Arithmetike, without whiche, Geometry can not Philos

diminiſhe. ſtande. And how needfull Arithmetike is to diminiſhe, it appeareth, ſeeyng ſo many doctors gathering ſo great miſteries out of no bꝛe, and ſo muche to wꝛyte of it. And yf I ſhuld go about to wꝛyte al the commodities of Arithmetike in ciuyle actes, as in gouernaunce of common weales in time of peace, and in due pꝛouyſion and oꝛdꝛe of armies, in tyme of warre: Fꝛo numbꝛyng of the hoſte, ſummynge of their wages, pꝛouiſions of vytayles, bewinge of artillerye, with other armour: Beſyde the connyngeſt poynt of all, fꝛo caſtynge of grounde, fꝛo encampynge of men, with other like. And how many waies alſo Arithmetike is conducible fꝛo al priuat weales, of Lꝛdes and al poſſeſſioners, of merchauntes and all other occuppyers, and generally, fꝛo all eſtates of men, beſides auditours, treaſoꝛers, receyuers, ſtewardes, baylyffes, and ſuch lyke whoſe, offyces wyth out Arythmetyke is nothynge. If I ſhuld (I ſay) particularly repete all ſuche commoditees of thys noble ſcience of Arythmetike, yt wer ynogh to make a very great boke. Sc. No, no ſyꝛ: you ſhal not need: fꝛo I doubte not, but this that you haue ſayde, were enough to perſwade any man, to thynke thys art to be ryght excellent and good, and ſo neceſſary fꝛo man, that (as I thynke nowe) ſo muche as a man lacketh of it, ſo muche he lacketh of hys ſence and wytte. M. What,

are

are you so farchaunged sens, by hearynge the fewe commodities in generall? By likely hode you wouldbe farre chaunged, if you knew all the commodities particular. Sch. I beseche you syr, reserue those commodytyes & rest yet behind, vnto their place more cõuenient. And if yee wyll be so good as to vtter at this time this excellẽt treasure, so that I may be somewhat entyched therby, & if euer I shallbe able, I wyl requite your paine. M. I am very glad of yout request, and I wyll do it speedily, syth that to learne it you be so ready. S. And I to your authoritye my witte doo subdewe, what so euer you saye, I take it for trewe. M. that is to muche, and meete for no man, to be beleued in al thinges, with out shewyng of reason. Though I might of my Scolar some credence require, yet except I shew reason, I do not it desyre. But nowe syth you are so earnestly sette this arte to attayn, best it is to omitte no tyme, lest some other passion coole thys great heat, and thẽ you leaue of before you fe thend. S. Thoughe many there be so vnconstant of mynde, that flytter and turn with euery wind, which often begyn and neuer come to the end, I am none of theyr sorte, as I truste you partelye knowe. For by my good wyl what I ones begyn, tyll I haue it fully ended, I wold neuer blynne. M. So haue I founde you hytherto in deed, and I trust you wyl increase

rather then go backe, For better it were neuer to assaye, than to shrinke and flee in the middle waye. But I truste you wyll not so do, therfore tell me briefely, what call you the science that you desire so greatly? S. Why syr, you knowe. M. That maketh no matter, I woulde heare whether you knowe, and therefore I aske you. For great rebuke it were, a science to haue studyed, and yet can not tell howe it is named.

S. Some call it Arsmetrike, and some Augrime. M. And what doeth those names betoken? S. That yf it please you, of you wolde I learne. M. Bothe names are corruptely writen, Arsmetrike for Arithmetike, as the Greekes call it, and Augrim for Algorisme as Arabians founde it, whyche bothe betoken the science of numbring. For Arithmos in Greke is called numbre, and of it cometh Arithmetike, the art of numbryng. So that Arithmetike is a science or art teaching the maner and vse of numbrynge, and maye be wrought diuersely, with penne or counters. But I wyll fyrst shewe the worckyng with the penne and then thother in order. S. tho. This I wil remembre. But how many thinges are to be learned, to attaine this arte fully? M. There are reckened cõmonly, vii. Numeration, Addition, Subtraction, Multiplication, Diuision, Progression, and Extraction of radicals: to these, some men adde

ἀριθμητικὴ.

ἀριθμὸς

adde Duplation, Triplation, and Mediation. But as for these last three, they are contained vnder the other vii. For Duplation and Triplation, are conteined vnder Multyplication, as it shal appeare in their place. And mediation is contayned vnder Diuision, as I wyll declare in his place also. Scho. Yet than there remayn the firste vii. kindes of numbryng. M. So there doothe: Howbeit, if I shall speake exactely of partes of numbryng, I must make but fiue of theym: For Progression is a compound operation of Addition, Multiplication and Diuision. And so is the extraction of Radicals. But it is no harme to name them as kyndes seuerall, seyng they appere to haue some seuerall woorkyng. For it forceth not so muche to contende for the numbre of them, as for the due knowledge and practisyng of them. S. Than you will that I shall name them as vii. kyndes distynct. But now I desyre you to instruct me in the vse of ech of them. Mai. So wyll I, but it must be done in ordre, for you may not learne the last as soone as the first, but you must learne them in that ordre as I dyd rehearse theim, yf you wyll learne them speedily and well. Scho. Euen as you please. Than to begin, Numeration is the first in order, what shall I do with it? May. First you must know what the thing is, and than after learne the vse of the same.



Numeration is the arte to expresse and read all sums proposed. and is of.ii. sorts, for eyther it geueth thereby ỹ valew of a sum proposed, eyther els it expresseth a sum cõceued by figures and places due. Schl. Why? than me thynketh you put a differẽce betweene the valew and the fygures. M. Yea, so do I: For the valew is one thyng, & the figures are an other thyng: and that commeth partly by the diuersity of fygures, but chyefly of the places, wherein they be sette. S. Than I must knowe here iii. thynges: the Value, the figure, and the Place. Mayst. Euen so: but yet adde Order to them, as the fowerth. And fyrst marke, that there are but x, figures, that are vsed in arithmetike, and of those x, one doth signify nothynge, whyche is made lyke an, o, and is called priuately a cyphar, though all the other somtyme be lykewyse named. The other ix, are called signifyng figures, and be thus fygured.

cyphar.

figures.

1 2 3 4 5 6 7 8 9

And this is their valew.

i. ii. iii. iiii. v. vi. vii. viii. ix.

But here must you marke, that euerye fy-

gure

gure hath two values: One alwaies certaine that it sygnyfieth properly, whych it hath of his forme. and the other uncertayne, which he taketh of his place.

¶ A place is called the seat or roume that a sygure standeth in. And looke how many sygures are wrytten in one sum, so manye places hath that whole valu. And the first place must be called that that is next to the right hande, and so rechenyng by ordre towarde the lefte hande, so that that place is last, that is nexte to the lefte hande. As for example, If there stood before you syxe men in a rowe. syde by syde, and you should tell them as they stande in ordre, begynnyng with the manne that were nerre to your ryght hand, than he that were nerre him, shoulde be called the seconde, and so forth to the farthest from your ryght hande, whych is the syxt and the last. ¶ A place

S. Sir I perceiue you wel, so might I recke letters or any other thynge. As yf I shulde wryte, viii. letters after this ordre, a, b, c, d, e, f, g, h, nowe muste I saye, h, is the syxte, g, the v, f the iiii, e the iii, d the ii, c the i, b the seuenth, and a, the viii.

M. That is well doone, And after the same sort vse hereafter, so that what I declare by one example, doo you expresse by an other, and so I shall perceyue whether you vnderstand it or no. And so passeouer nothyng, tyl you perceiue it well and be expert therin.

Scho:

S. Syr, I praie you, how many of these places be there in all? M. There is no certayne nũbre of them, but they ar somtymes more and sometymes fewer, accordynge to the summe that is expressed. For so many as the figures are, so many are the places: and the last place is so called, not bicause it is the last of all other, but it is the last of that present summe, and it maye be the myddle place in an other summe. S. Me seemeth I perceiue this very well, as touchyng the order of reckenyng of the places: But as for the nombre of theim, you say there is no certaynte. Now there resteth to declare the valu of the figures, by diuersitie of places, whiche you called, The valewe vncertayn. M. But fyrst lette me heare whether you knowe perfectly the certayne valewe. Scho. Yes syr, as you wrote theym, so I marked them. M. Howe write you then. vi? By this figure. 5. M. And how. vij. S. Thus. 6. M. Wrote these iiij. noumbres eche by it selfe, as I speake them. viij. iiij. iij. Scho. 7. 4. 3. M. Howe write you these foure other. ii. i. ix. viii? Sco. Thus (I trowe) 2. 1. 6. 8. M. Nay, there you myste. Loke on mine example again. S. Syr trouth it is, I was to blame, I toke 6 for 9, but I wyll bee warer hereafter. M. Nowe then take hede, these certayn values euery figure representeth, when it is alone writē, without other figures ioined to him.

Value vncertayne.

And

And also whē it is in the fyrst place, though many other do folow, as for example. This figure 9 is. ix. standing now alone. S. Howe is he alone, and standeth in the myddle of so manye letters? M. The letters are none of his felowes. And if you were in Fraunce in the myddle of M. Frenche menne, if there were no Englysh man with you, you wolde recken youre selfe to bee alone. S. Só it is. Then 9 wythout more fygures of Awgrym, betokeneth nyne, whatsoeuer other letters be about it. M. Euen so, and so dothe it, if it be in the fyrst place ioyned with other, howe many so euer do folowe, as in this example 3679: you see 9 in the first place, and dooth betoken, ix. as if he wer alone. S. I perceiue that. And doth not 7, stande in the seconde place, and betoken, vii? and 6 in the thyrde place, and betoken vi? And so 3 in the fourth place, and betoken thre? M. Their places be as you haue sayd, but their valewes are not so. For as in the fyrste place euerye fygure betokeneth his owne valew certaine onely, so in the seconde place euerye figure betokeneth his owne valew certayne, x, tymes: as in that example, 7 in the second place, is vii. tymes x, that is, lxx. And in the thyrde place, euery figure betokeneth his owne valewe a hundreth times, so that 6 in that place betokeneth, vi, hundred. And in the iiii. place euery fygure betokeneth his owne

valewe

valeiue a. M. tymes, as in the forelayde numbre: 3 in in the fourth place standeth for iij. M. And in the fifth place, euery fygure standeth for his owne value x. M. tymes. And in the. vj. place, a. C. M. tymes. And in the. vij. place, a M. M. tymes. And in the viij. place, x. M. M. tymes: so that euery place exceedeth the former ten tymes. &c. As thus, if I make this nombre at all aduentures, 91359684, here are. viij. places. In the fyrst place is 4, and betokeneth but foure. In the seconde place is 8, and betokeneth. x. tymes eyghte, that is lxxx. In the thyrde place is 6, and betokeneth. vi. C. In the fourthe place 9, is. ix. M. And 5. in the v. place, is x. M. tymes, 5. & is l. M. So 3. in the. vi. place, is a C. M. tymes, 3. that is. CCC. M. Then 1 in the. vii. place, a M. M. And 9 in the viij. place x. M. M. tymes 9, that is. xc. M. M. But nowe I can not easyly nor quickly reade it in order. M. That shall you practise by this meanes, Fyrst put a pryche ouer the fourthe fygure, and so ouer the. vii. And yf you had so many, ouer the x. xiii. xvi. and so forthe, styll leauynge two fygures betweene eche two pryches. And those roumes betweene the pryches, are called, ternaries. Then begyn at the laste pryche, and see howe many fygures are betwene hym and the ende, whiche can not passe thre, reckenyng hym selfe for

many ternaries.

for one, than pronounce theym. as yf they were written alone, from the reste, and adde at the ende of theyr valewe so many tymes thousande, as your noumbre hath prickes. Then comme to the nexte. iii. fygures, and founde them as yf they were apart from the reste, and adde to their value so many tymes thousandes, as there are prickes betweene theym and the fyrste place of youre whole numbre. And so doo by euery other iii. figures folowing, if you haue mo, as in example, 9.359684, this was youre noumbre: put a pricke ouer 9 in the fourth place, and ouer 3 in the. vii. place, and then no more, (for your places come not to tenne) as thus 9.359684. Now go to the last prick ouer 3, & take it, and the figure 9 that foloweth it, and valew them alone. S. 93 that is xci. M. So is it: but than adde for the noumbre of your prickes twyse M. Scho. That is. xci. thousande thousande. M. So is it. Nowe take the. iii. other figures from 3 to the next pricke, and valewe theim. Sco. 359, that is CCC.lix. M. Nowe adde for the one prick that is betwene them and the fiest place M. S. CCClix. thousande. M. Nowe come to the other. iii. fygures that remayn. S. 684. that is. vi. C. lxxxiiii. M. Nowe haue you valewed all. And at the end of this last nũbre, you shall adde nothyng, because there remayneth no prycke nor numbre after it:

yet

Yet proue in an other numbre, as thus.

230864089105340.

Scholer. 230864089105340. I haue pricked them as you taughte me, but I am in doubt, whether I haue doon wel or no, because of the cyphers: For I remembre, you tolde me that they do signifye nothyng, and therfore I doubt whether I shuld reckon them for a figure in settyng of the prickes: and againe I knowe not wherfore they serue. M. That wil I tel you now: In deed they ar of no valewe them selfe, but they serue to make vp number of places, and so maketh the figure folowyng them to be in a further place and therfore to signifie the more value: as in this example, 90, the cypher is of no value, but yet he occupieth the first place, and causeth 9 to be in the second place, and so to signifye x. tymes 9, that is xc. so that ij. cyphers thrusteth the figure folowing them into the iij. place, and so forth. Schol. Than I perceaue in the example aboue, I haue pricked wel ynough: for though that cipher that is pricked, signify nothyng, yet must he haue the prycke, bicause he came in the iij. place. Than wyll I proue to numbre that summe. First there is 230. M, M, M, M, 864, M, M, M. And what shall I nowe doe? There is a cypher in the iij. place, and no figure after hym, but they that I haue reckened. M. Ye dydde serue for theym that

you

you haue all ready reckened, to make them in a place further then they shoulde be, if he were away, and therfore now you shall lette hym go. And so do alwayes when he occupyeth the place next before any pricke, whiche is the laste of that ternary: and a cypher in the last place doeth nothynge. S. Then I shall take but 89. M. M. No, but goe forthe. S. 105. M. Nowe are all my prickes spent, and yet remayn 340, so I must valewe the. CCC. xl. only. Mayst. Now can you reckon after this sorte. And remembre, that euery such rounde so parted, is called a Ternary or Trinitie. Some doo part suche great numbres with letters after this maner:

c b a c b a c b a c b a c b a

2 3 0 8 6 4 0 8 9 1 0 5 3 4 0. In which example ye may se that, a, supplieth the rounde of your pricke. And some doeth part the numbres with lynes after this forme, 230|864|089|105|340. where you se as many lines as you made prickes, and to one entent, saue that the lynes doeth more playnely parte euery .iij. figures, accordyng as they should be valewed vnder one denomination S. Yea syr, but yf you shulde shewe me a numbre so parted, I shulde take it for many numbres, and not for one. M. So myght you do, not knowyng my meanyng. But what yf I did set forth the numbre without lynes, (youre selfe (for the ease of reckenyng) dyd so parte

it with lines, wold you forget wherfore you did it, and then take them for many nũbres? &c. No, I trowe not, but yet I doubte. M. Then vse that, that you lyke best, for all the thre waies is to one entent, saue (as I sayd) that the lynes doeth more plainly distincte the denominations. S. What call you denominations? Mai. It is the laste valewe or name added to any summe. As when I saye. C. Crvii. poundes: poundes, is the denomination: And lyke waies in sayyng: xij. men, men, is the denomination: and so of other. But in this place (that I spake of before) the last numbre of euery Ternary, is the denominatiõ of it. As of the first ternary, the denomination is vnites: and of the second ternary, the denomination is thousandes, & of the thirde ternary, thousande thousandes, or mylliõs: of the. iiij, thousande thousand thousandes, or thousand millions, and so forth. S. And what shall I call the value of the. iii. figures that maie be pronounced before the denominatours, as in sayenge: 203.000.000, that is, CC. iii. mylliõs, I perceiue (by your woordes) that mylliõs is the denomination, but what shal I call the. CC. iij. ioyned before the mylliõs? M. That is called the numeratour or valewer, and the hole summe that resulteth of them both, is called the summe, value, or nũbre. S. Now is there any thyng els to be learned in Nume-

ration?

nomina.

meras.

mme.

cation? or els haue I learned it fully? Ma. I myght here shewe you, who were the first inuentours of this arte, and the reasons of all these thynges that I haue taughte you, as why you shulde recken your order of places backward, I meane from the ryght side towardes the left, with many other thynges, touchynge the causes and reasons of it, but that will I reserue till ye haue lerned ouer all the practise of this art, least I shoulde trouble your wytte with ouer many thynges at the fyrst. But yet this must you marke, that there are thre kyndes of numbre: one called digettes: an other articles: and the thyrde myrte numbre. A diget is any numbre vnder 10. as this: 1, 2, 3, 4, 5, 6, 7, 8, 9. And 10 with all other that may be deuided into 2. partes iuste, and nothyng remayne, are called articles, suche are 10, 20, 30, 40, 50, &c. 100, 200, &c. 1000. &c. And that numbre is called myrte that conteyneth articles, or at the least one article, and a digette, as 12, 16, 19, 21, 38, 107, 1005, and so forthe. And for the more ease of vnderstādyng and remembrance, marke this: The diget numbre is neuer writē with more thā one figure, but the article and the myrt numbre at euer writē with more thā one figure: & thus they differ, ỹ the article hath euermore this cyphre 0 in his first place, & ỹ myrt numbre hath euer there some diget. S. By these last wordes,

Thre kyndes of numbre.

Diget.

Myrte.

Article.

I perceiue much better thā I dyd before, & nowe (I thinke) I wil neuer misse to know those thyrce asunder. Mai. If you remembre now all that I haue sayd, you haue learned sufficiently this fyrst kynde of Arithmetike, called Numeration. Howbeit I will yet exhorte you nowe to remembre bothe this that I haue sayde, and all that I shall saye, and to exercise your selfe in the practise of it: for rules without practise, is but a light knowledge, and practise it is, that maketh men perfect and prompte in all thynges. And as you haue learned to gather the valewe of a summe proposed, so must you practise to expresse any summe with theyr dewe fygures: as for a prefe: How expres you this summe, fiue thousand, two hundred, fyfty and seauen? S. This troubleth me nowe, whether I should begyn at the fyrst figure or at the laste. For reason (me thinketh) should cause me to begyn at the fyrst, and yet yf I write it as you spake it, I muste begyn at the last. M. When you know your places perfectely, you may begynne where you lyste. But the more ease for your hande is, to begyn with the last, that is to say, as I did speake them. But for the more surety, a while you may begyn with the fyrst, repeting my wordes backwarde, thus: Seuen, fyftye, two hundred, fyue thousande: or els soundyng them all by their digit or valewer, as thus: Seuen, fyue, two

two, fiue: for that way is easyest. But than must ye looke well, whether there be any cypher in your summe, that he may be serue in his place. As if your laste valewer of youre summe (as you speake it) bee aboue 9, than is there a cypher in the fyrst place. And if it be a hundred or aboue, than is there ii. cyphers, one in the first place, and an other in the seconde, and so forth. But because thys thyng is suche that can not be set forth without many wordes, I thynke best here now at the ende of Numeration, to adde a table easy and ready for the first exercyse of it. Lo this is that table.



The leuelyde of hande.

¶ This table (as you maie se) hath a leuen places, and in eche of them are set all the digettes, whose certayne valewe is wrytten on the right hande of the table, and the valewe vncertaine on the lefte hande. So that by this

thys table you may learne both howe to expresse any numbre that you lyste (yf that it exceede not. xi. places) that is to saye, lyttle thousand myll.yons, and so may you by the healpe of it, valew all summes proposed vnder the sayd noumbre, as for ensample. The summe that I proposed before, whych was fiue thousand, two hundred, fifty and seuen, yf you will expresse it, take the fyrst numbre (as I speake it) which is fiue M. whose valowre or treasyn valow is v. & his vncertain valowe or denomination is M. Fyrst you shall seeke at the right hand the valewer, v. Than seeke alonge vnder the titell of denominations, towarde the lefte hande, tyll ye fynde thousandes, and vnder it right at the foote of the table, is the numbre of the place that is the fowerth, wherein you muste wryte your digite or valower, v. Than come to the seconde parte of the numbre ii. hundred, whose valewer is 2, and denomination C. Seeke it. at the rygbte hande of the table, and go alonge vnder the denominations, towarde the lefte hande, tyll you come vnder C. than looke to the foote of the table, and there shall you see the noumbre of the place that is to saye, iii, wherein you muste set your digite 2. Than doo so by your other two noumbres that remayne, and you shall fynd fyue in the seconde place for your fyfty, and 7 in the fyrste place for your vii. And thus

maye you do with other numbres. S. Mayster I thanke you harrelye. I perceiue you seeke to instruct me moste plainly and briefly, and not to hyde your knowledge with subtyll woordes, as many doe. For this rule is so plaine, that I can desyre it no playner. And thoughe it seem somewhat long, yet I perceiue it a sure waye. May. So is it, and though it be long, yet it is neyther to longe, neyther to playne for yonge learners that lacke practise: for this table is in steede of a teacher, to them that lacke one. But now I truste I haue sayd ynough of Numeration whyth, after you haue well practised, then may you learne forth. Sco. Syr in numeration I haue well practised, and am ready to learne forth. May. That is well, but what should you next learne can you tell? Sco. I remembre you sayd, that Addition was next. M. Euen so, and what that is must you first knowe.

ADDITION.



Addicion is the reduction and bringing of two sūmes or more into one: as if I haue 160 bookes in the latine tongue, and 136 in the greke tongue, and would knowe howe many they bee in all. I

muste wryte those, ii, noumbres, one ouer an other, wryting the greatest numbre highest, so that the firste figure of the one, bee vnder the firste figure of the other: And the seconde vnder the seconde, and so forthe ouerthwarte. Whe you haue so done, drawe vnder them a straite line, then wil they stand thus

160
136

Nowe begynne at the firste places towarde the right hande alwayes, and put togither the ii, first figures of those two summes, and loke what cometh of them, wryte vnder them, right vnder the lyne: As in sayyng, 6 and 0 is 6: Wryte 6 vnder 6, as thus,

160
136
6

And then go to the seconde figures, and do likewaies: as in sayyng, 3, and 6 ys 9, wryte 9, vnder 6 and 3, as here you see.

160
136
96

And lyke waies doo you with the figures that been in the third place, saying, 1 and 1 be 2: wryte two vnder them, & then wil your hole sum apere thus.

160
136
296

So that nowe you see, that 160 and 136 do make in all, 296. Ech. What this is very easye to doo. me thynketh I can do it euen sythe. There came throughe Cheapes side, two droues of cattell, in the firste was 848 sheepe, and in the seconde was 186 other beastes, those two summes I muste wryte as you taughte me thus.

848
186

Than if I putte the. twoo fyrste fygures togither; sayinge: 6 and 8, they make 14. That must I write vnder 6 and 8 thus, M. Not so, and here are you twyse deceaued, fyrste, in goyng aboute to adde togither ii, summes of sundrye thynges, whych you ought not to doo except you seek only the numbre of them, s care not for the thinges. For the same that shuld result of that addition, should be a sum nother of sheep nor other beastes, but a confused sum of both. Howbeit, some times you shall haue summes of dyuers denominations to be added, of whych I will tell you anone: but fyrst I wyll shew you, wher you were deceiued in an other ponte, and that was in wryting 14, whyche came of 6 and 8, vnder 6 and 8, whiche is vnpossible: For how can two figures of two places, be wrytten vnder one fygure and one place? S. Tru the it is: but yet I dyd so vnderstand you. M. I sayde in deede that you shoulde write that vnder them, that dyd result of theym both togyther, whych sayinge is alwayes trewe, if that summe do not exceede a diget. But yf it be a myxt numbre, then must you wrvte the dyget of it vnder your fygures, as I haue sayd before: but and if it bee an artycle, than wrvte o vnder theym, and keepe the artycle in your mind, And therfore when you haue added your second figures, whych

1 8 6	
1 4	

occupy

occupye the place of tennes, you shall putte that vnto, which you kepe in your mynde: for though it were no in deede, yet in that place it is but as 1, because that euery place of that place is ten, for it is the place of tennes. And in lyke maner, yf you haue in the seconde place so great a numbre, that it amounteth aboue 9, than write the digetes, and reserue the article in your mynde, euer adding it to the nexte place folowynge. and so of all other places, howe manye so euer you haue. And if you haue a myxte numbre, when you haue added your laste figures, the write the digite vnder the laste figures, and the article in the nexte place beyonde them: so shall your numbre resulting of addition, haue one place more than the noumbres whyche you should adde togyther. S. Nowe doo I perceaue you, and the reason of this is (as I vnderstande) because, that no one place can containe aboue 9, whyche is the greatest figure that is, and then all tennes or articles must be put to the nexte place folowynge. for euery place (as I may se) exceedeth the other place nexte before hym, by 10. Nowe (if it please you) I wyll returne to my exaumple of carte. But I remembre you sayd, I might not adde summes of sundrye thynges togyther, and that might I see by reason. Mayster. Trouth it is, yf you seeke the summe of any thynges, but if you only seke a sum, y haue no

X place.

respect

respect to the thynge, than were it better to name the summe onely without any thyng as in saying 848, without namyng thyse or any thyng els: And likewayse 186 namyng nothyng. Now lette me see, how canne you adde those two summes. Scho. I must fyrste set them so, that the twoo fyrst figures stande one ouer an other, and so the other erbe one ouer hys felow of the same place, than draw a lyne vnder them bothe. And so like wayes of other fygures, settyng alwayes the greatest numbre hygheste, thus.

848
186

Than muste I adde 6 to 8, whyche maketh 14, that is a myxte numbre, therfore muste I take the vygent which is 4, and wryte it vnder 6 and 8, kepyng the article 1 in my mynde, thus.

848
186
4

Than doo I comme to the second figures, addyng theym togyther, saynge: 8 and 4, make 12, to whyche I putte the 1 reserued in my mynde, and that maketh 13, of whyche numbre I wryte the vygete 3 vnder 8 and 4, and keepe the article in my mynde, thus.

848
186
34

Than come I to the thyrd figures, sayinge, 1 and 8 make 9, and 1 in my mynde maketh 10. Wyr shall I wryte the cyphar vnder 1 and 8? M. Pea. Scholar. Than of 10 I wryte the cyphar vnder 1 and 8, and keepe the article in my mynde. Mai. What need that, seeyng there folowyth no

more

more figures? S. Syr I had forgotten, but I wyll remembre better hereafter. Than seyng I am come to the last fygures, I muste wryte the cypher vnder them, and the article in a farther place after the cyphar, thus.

348
186
1034

Maist. So now ye see, that of 148, and 186 added togyther, there amounteth 1034. S. Now I thynke I am perfecte in Addition. M a. That wyll I proue by this exaumple. There are .ij. armyes of souldioures, in the one are 106800, and in the other 9400: Howe many are there in bothe armies, saye you? S. Fyrst I set them one ouer an other, begynnyng with the fyrste noumbres at the eyght hande, thus.

106800
9400

But the nether numbre wyll not match the ouer numbre. M. That forceth not. S. Than doo I adde 0 to 0, and there amounteth 0, that muste I wryte vnder the fyrst place, thus.

106800
9400
0

M. Well sayd. S. Then lyke wayes in the seconde place I adde 0 to 0, and there ariseth 0, whyche I wryte vnder the second place, thus.

106800
9400
00

Than I come to the thyrde place, sayeng: 4 and 8 make 12, of whyche I wryte the diget 2, and kepe the article 1 in my mynde, thus.

106800
9400
200

Then adde I 9 to 6, which maketh 15, to that I adde the arti-

cle

cle, 1, that was in my mynde, and it is 16, I wrote 6 vnder 6 and 9, and kepe 1 in my mynde, thus.

M. Why do you not wryte both figures, seyng you are come to the laste couple of numbres? S. Naye, reason the weth me, that I muste adde that article that is in my mynde, vnto the nexte figure of the ouer summe, though there be no more in the nether summe. M. That is well considered, then doo so. S. Then saye I, 7 in the ouer summe, and 1 in my mynde, make 1: that I wrote vnder 0. Then folowth there 2 in the ouer summe, whiche hath none to be added to it, for there is none in the nether summe nor yet in my mynde, therefore I thynke I muste wryte that euen as it is. M. Yea. S. Then dooth my hole summe appere thus.

	106800
	9400
	116200

M. If you marke this, you haue learned perfectly the comon addition of all summes whiche are of one denomination, so that you obserue this also, that in Addition you must haue .ij. numbres at the leaste, or elles howe can you saye that you dooe adde? And euer lette the greatest numbre be wryten hyghest, for that is the beste way, though it be not necessary. And forget not this, that yf you haue many numbres to adde togyther, you shal haue oftentymes an article of a greater value then

10: sometymes 20, sometymes 30, sometimes more, yea peraduenture 100: Therefore as you dyd with the article 10, so do with them, reseruyng them in your mynde, and addyng to the numbre nexte folowynge so many, as their valewe or valewe certayne is, that is to say, 2 for 20, 3 for 30, and so forth of other. But if the article bee 100, then muste you not adde the article to the nexte figures folowynge, but to the thyrde figures frome them, as I wyll shewe you anone by exaumple. And if it chaunce the numbre to be such that it doo comprehend two sundry articles (that is one of tennes, and an other of hundredes) then muste you reserue theym bothe in your mynde, and adde the article of tennes to the fygures that folowe next, and the article of hundredes to the figure of the thyrde place frome thence.

Nowe take this exaumple for all. I wolde adde these. xiij. summes in one, which I set after this maner. Then do I begyn and gether the summe of the fyrste fygures whiche commeth to 107. For first I take 9 there .x. times, & that is 90: then 9 and 8 is 17 that is in al 107: Of which sume I write the 7 vnder the fyrste figures, and than haue I an article of 100 in my mynde, whyche other I muste

	4	8	8 9
	4	5	9 9
	2	2	9 9
	3	6	9 9
	2	3	9 9
	4	0	9 0
	1	0	9 9
	3	1	9 8
		2	9 9
		6	9 9
		8	9 9
		4	9 9
		3	8 9

I must kepe in my mynde, tyl I come to the thyrde figures, whiche are in the roumes of hundredes, or elles I maye for feare of forgettyng, wryte one vnder the thyrde row of figures, makynge two lynes, as you see here doone. And then muste I wryte the digytes vnder the lowest lyne, and this is the surest waye, when the summe is so great, that the addition of one rowe passeth 100. When I haue so done, I must thā come to the second row of figures, and adde them togither, whiche dooth make 115, of which summe I wryte the digit 5 vnder the same second rowe, and then I haue an article remaynyng of two fygures, of whiche the one (that standeth for 10) must be added to the second or next place after them, that I dyd laste adde. And the other (that standeth for 100) must bee addded to the thyrde place from thense. Scho. That is to saye, the fourthe place from the fyrst lyne or rowe of fygures. Mayst. Euen so. And then wyll the summe appere thus. Then adde the thyrd rowe of figures with the two vnitees betweene the lynes, and the summe amounteth to 50, of whyche I wryte the cyphre vnder the same

	4	8	8	9
	4	5	9	9
	2	2	9	9
	3	6	9	9
	2	3	9	9
	4	0	9	9
	1	0	9	9
	3	1	9	8
	2	9	9	
	6	9	9	
	8	9	9	
	4	9	9	
	3	8	9	
	1	1		
	1			
		5	7	

thyrde

thirde rowe, and the 5 vnder the nexte figures towarde the lefte hande. Then I adde the figures of the fourthe rowe with the 1 and 5, that are vnder: theym betweene the twoo lynes, and they make 29: then write the 9 (that is the digeste) vnder the fourthe place, and the 2, that is the article beyond it towarde the lefte hande. So those summes do make 29057. S. Thys seemeth somewhat hard, by the reason of so many numbres together. Howe be it I thynke if I doo often proue euen with this same example, I shall be able to do so shortly with any other sume. M. So shal you: For it is often practise that maketh a man quicke and ripe in all thynges: But breasle of such great summes, in whych there may chaunce to be some errour I wyl teache you how you shall proue whether you haue done well or no. S. That were a great helpe and ease. M. Begyn first with the highest numbre, and thā al the other orderly, and adde theym together, not hauing regarde to their places, but as though they were all vnites, and styll as your numbre increaseth aboue 9, cast awaye 9. Then go foorthe, euer castynge awaye 9, as often as it amounteth therto: And so do till you haue gone ouer all the numbres that you entended styll to add, and what soeuer remaineth after suche addition and castynge awaye of 9, write it in some voide place, by the vnde-

9
488
459
229
369
239
409
109
319
29
69
89
49
38
11
51
2905

of a lyne, for the better remembraunce, and then put together the fygures that resulte of the addition, styll castynge awaye 9 also. And then that, that remayneth, wryte at the other ende of that line: and if thole twoo be lyke, then haue you well done, but if they be vnlike, then haue you milled. As for example in this presente summe: The fyrst figure of the ouer lyne is 9, lette hym go: then 8, and 8 is 16, take away 9, and there remayneth 7, adde to it 4 that foloweth, and that maketh 11, from which if you take 9, there resteth 2. Then come to the nexte rowe, whose fyrst and seconde numbre are 9, therfore ouerpasse them both, and take the 5 to the 2, whych dyd remayne in the fyrst rowe, that maketh 7, put therto the 4 folowyng, that maketh 11, thence take 9, and there remaineth 2. Then go to the thyrd line, whose two fyrst numbres you may let passe, because they are nines: then take the two 2, whiche with the other two that remained in the seconde row, make 6. Then go to the foweth rowe, whose two fyrst numbres let go, & take the 6 to the 6 that remained, and that maketh 12: take awaye 9, and there resteth 3, whiche with the 3 that is next, maketh 6: And so go through all the other noumbres, you shall fynde that there remayneth 5 after you taste awaye 9 as often as you fynde yt, therefore wryte 5 at one ende of a lyne

in a voyde place thus. 5 ——— Then gather all the figures of the totall summe which is vnder the lowest lyne, and caste awaye 9 as often as you fynde it, as thus: 7 and 5 make 12, take away nine, and there resteth 3, to that if you add the 2 that is last (for you may let go the 9) then both it make 5, whych you muste write at the other ende of the lyne that you made in the voyd place, and it wyll be thus, 5 ——— 5. And than you se that those twoo fygures be like, wherby you maye know that you haue done well, and so may you proue in any other.

Scho. If it please you, I wyll proue in an other summe. Maist. With a good wyll. Sco. Then wyll I take one of youre former examples, whiche was thys.

Fyrst in the highest lyne 8 and	106800
6 make 14. than 9 taken away	6400
remayne 5; to whyche I adde	116200
the 1 that foloweth, and that	
maketh 6. Then come I to the second lyne,	

where I fynde firste 4, whiche with 6, maketh 10, from that I take 9, & there resteth 1, the next figure, is 9. and therfore I let him alone, so fynde I one remaining, whyche I set at the ende of a lyne thus, 1 —— Than I come to the totall summe, and there I fynd y all the fygures put togyther, make 10, frō which I take 9, & there resteth 1 also, which I put at the other end of the lyne thus 1—1.

And bycause they be lyke, I knowe that I haue well added. May. So you knowe nowe bothe howe to adde two summes or more together: And also how to proue whether you haue done well or no: whych thyng also you may doo by Subtraction. But because you can not yet skyll of it, I wyll lette that passe tyll anon, and wyll teache you nowe howe to adde sūmes of dyuers denominations: whyche thyng can neuer be, but when the one denomination is suche, that it conteineth the other certayne tymes. And yet you shal adde them to the other, not after this sorte as you dydde them that were of one denomination but after suche a sorte, as I wyl nowe shewe you, that is to say. If you haue a summe of diuers denominations, then loke that ye set euery denomination by him selfe with some note or figure of hys denomination as they be wonte to be written: Then wryte your other summes so vnder that firste, that euery one be set vnder the other of the same denominations. As for exaumple: If your denominations bee poundes, shillinges, and pens, wryte poundes vnder poundes, shyllynges vnder shyllinges, and pens vnder penys: and not shyllynges vnder pens, nor pens vnder pounds. S. Now that you haue spoken it, me thinketh it needed not to warne me of it, for it were agaynst reason so to confounde summes: but yet if you had not spo-

ken

ken of it, peraduenture I should haue bene deceaued in it. M. If you doo saye it is so playn, I wyll speake no more of it, but with an example make the matter to appere euidently. If vnto one man oweth me 22 li, 6 ß 8 d. An other oweth me 5. li. 16 ß 6 d. And an other oweth me 4. li. 3 ß, I wolde knowe what this is all togyther. Therfore must I fyrst set downe my greatest summe, & then the other, euery one vnder his denomination, agreynge to the greatest summe, as thus.

li.	ß.	d.
22	6	8
5	16	6
4	3	

Then muste I begynne at the smallest numbres which muste always be sette next the ryght hand, and adde them togyther. And if the summe of them wyll make one of the nexte denomination, then must I kepe it in my mynde, tyll I come to that place: or els for more easynes, write it vnder that place betwene the double lyne, & vnder that first place muste I note the residewe, if there remayn any of the same denomination, but yf there remayne none, then nede I to write vnder it nothyng. And this is all that you must marke in this addition: for al other thynges are lyke to the other maner of addition before mencioned. Therfore the chiefest poynte of this addition is, to knowe the valewes of common coynes, & rated summes: As how many shillyngs be in

a pound, how many pewies in a shilling. Of whiche and of other like thynges, I wil instruct you hereafter, in teachyng of Reduction. But now I may not disturbe your wit from the thyng that we ar about: Therfore let vs returne to that former example, whyche I proposed of .iii. letters, whiche summes when I had set ordrely, they stode thus with a double lyne vnder them.

li.	s.	d.
2	6	8
5	16	6
4	3	

Then to adde the vnto one summe, muste I begyn at the right hande, where the smallest denomination is, and adde them togyther first, sayeng: 6 & 8 make 14, nowe seing there ar pentes, & that 12 d. make one shilyng, whiche is the next valewer, I take away 12 from 14, and there resteth 2, which I write vnder the pentes, and for the other 12, whyche make 1 shilyng, I write 1 vnder the title of shilynges thus.

li.	s.	d.
2	6	8
5	16	6
4	3	
	1	

Than doo I adde all the shilynges togyther, and find them 25, to whyche I adde the 1 betweene the two lynes, & that maketh 26, but because that 20 s. do make 1 pound, I take away 20 from 26, and for that 20, I write 1 vnder the poundes betweene the two lynes, and the other 6 that remayneth, I write vnder the shillynges thus.

li. s.

Then come I to ẏ pounds addyng theim all togyther, and fynde theim to bee 32, therto I adde the 1 betwen the two lynes, and that maketh 32, why the summe I wryte downe hole, bycause there is no greater denominacion. And then my whole sum appeareth thus.

	li.	s.	d.
	22	6	8
	5	16	6
	4	3	
	1	1	
		6	2

So is my totall summe 32, li. 6. s. 2 d. And thus maye you proue in an other lyke summe. S. Then wil I call the hole charge of one monethes commons at Oxford with battelyng also M. So to, let me see how you can doo. S. One wekes cõmons was 11 d. ob. q. and my battelyng that weke was 2. d. ẽ. q. The seconde wekes cõmens was 12 d. and my batlyng 3. d. The third wekes cõmõs 10. d. ob. ẽ my batlyng 2 d. q. c. The fourth wekes cõmons 11 d q, ẽ my batlyng 2 d ob. c. These 4 sũmes wold I adde into one hole sũme, ẽ therfore I wyl set them one ouer an other, thus. But I had forgotẽ I shuld haue set the greatest sum highest M. So is it cõmonly best, howbe it here it forceth not, ẽ in sũmes as this is, that go by order of wekes, daies, or yeres it is

d.		
11	ob.	q
2		q. q
12		
3		
10	ob.	
2	q.	c.
11	q	
1	ob	c.

q is better to kepe that order than to alter them, and to set the greatest nũbre hyghest, for that serueth for suche summes as go not by order. S. Then yt I haue sette them well ynoughe, I will begyn to adde them thus.

℔	
1 1	ob. q̃.
2	q̃. q.
1 2	
3	
1 0	ob.
2	q. c.
1 1	q̃.
1	ob. c.
	q

Fyrste of the smallest valewers at the right hand, whi che are called ceps, I fynde 2, and sepng that 2 ceps do make one q, I wyll wrte nothynge vnder the ceps, but will wrte 1 q, for 2 ceps vnder the kewes betweene the lynes, after this maner Than come I to the nexte valewers, where I fynd 2 q, and to them I adde the q, that is betwen the lynes, and so are they 3 q, but because 2 q. maketh one q̃, I wrte one q̃ vnder the farthynges betweene the lynes, and the q that remaineth, must I wrte benethe the nethermost line, vnder the kewes, thus.

℔	
1 1	ob. q̃. q.
2	q̃. q.
1 2	
3	
1 0	ob.
2	q. c.
1 1	q̃.
1	ob. c.
	q̃. q.
	q.

Then com I to the farthynges where I fynde 3, and the other q̃ that is betwene the lynes maketh 4 q̃: And because 4 q̃ make iust 1 peny. I shall wrte nothyng vnder the farthynges, but muste wrte 1 vnder the pens bee-

twene

twene the lynes. Than muste I adde the halfpence togither, of whiche there are 3, but seyng that 2 ob. make 1 ð, I must write 1 vnder the pēs betwene the lines: but how shall I doo it, for there is 1 already? M. Haue you forgotten how I dyd in addition of the great summe before? you muste set it vnder the other, so shall they both stand for 2: for yf you shuld set it before or behynd the other, they shuld make 11. S. I remembre it now, and I perceaue the reason. Then I will write 1 ob. vnder the halfpens, and for the other two halfpens whiche make 1 ð, I write 1 vnder the pens: Then come I to the pens, and synde that there are of them 52, than put I to them the 2 betwene the lynes, and that maketh 54. whiche amounteth to 4 s̄ 6 ð, the 6 ð, I must write vnder the pens, and the 4 s̄, I muste sette (I suppose) farther towarde the left hande by them self.

M. Euen so. S. Than appereth al my additiō, thus.

And the summe is 4 s̄ 6 ð ob. q. M. Nowe haue you doon this wel. But tel me, why dyd you wrote rewe, cee, thus q, c, and not rather thus q^c, as the fashion is. Scho. Because I thought it was the best way

1	2	ob. q.
	2	q. q.
1	2	
	3	
1	0	ob.
	2	q. c.
1	1	q.
	1	ob c.
	1	q. q.
	1	
s̄	6	ob. q.

way for due gatheryng of euery denomination by him selfe. M. So was it in dede. Well now, can you tell howe to proue this addition, and such other like of diuers denominations, to trie whether you haue doone well or no? S. I would I couldde. M. That shall ye doo by thys meanes: Fyrst as you dydde begyn to adde, so reckon agayn euery denomination by it selfe, and when you fynde so many small, that doo make any other denomination let theym go, and kepe in mynde onely the residue that wyll make no greater denomination, and loke whether there be any suche lyke vale we vnder the nether lyne, and yf there be, you haue wel done, and so go forth frome one denomination to an other, vnto the ende. But here must you note, that in gatheryng of the summes ye muste reckon those figures that are written betweene the lines with them that are written about theym, as for an example. I wyll examyne that summe that I dyd laste adde, whyche stode thus.

li.	s.	d.
2	2	6
5	16	6
4	3	
32	6	2

Fyrst I find 6 and 8 whyche maketh 14, from whiche I take 12, because it maketh one of the nexte denomination, and there remaineth 2, and vnder that place I see a lyke figure, therfore I know that to be well done. Then come I to the 5. where I fynde 3,

16 & 6, ꝑ maketh 26, I caste away 20, for they make an other denominatiō, that is to say, poūdes, & the 6 which remaineth, is like to the 6 ꝑ is write vnder thē euenly ꝑ lowest line, therfore that is wel don also. And thens I go to poundes, where I fynd 1, 4, 5, 22, ꝑ is 32, to whiche sum agreeth an other like vnder it. Therfore I iudge all well done. 8. I perceaue reason in this probation, nowe will I attempte the same in the summe that I dyd adde, whiche whan I had ended addynge, stoode thus. ō.

fyrst amongest the rees	1	1 ob.	q̄
I fynde but 2, whyche		2	q̄.q.
make 1 q, euen, therfore		1 2	
there must nothynge bee		3	
vnder theyne for them:		1 0 ob.	
and amongest the newes		2	q.c.
I fynde, 3, of whyche 2		1 1	q̄.
make 1 q̄, therfore I lette		1 ob.	c.
theim go, and the one q		1	q̄.q
that is lefte, hathe an o-		1	
ther like vnder his place, &			
therfore that is wel don.		4 6 ob.	q.

Thā the farthings are full 4, which make 1 ō, & therfore I let the go. Amongest the halfpēs ther is one odde, for 2 must I caste away, bicause they make 1 ō, & vnto it answereth a like sum vnder it. The pēs are 54, frō whych I take awai 48, ꝑ make 4.s. & ꝑ 6 remainig agre to a like figure set vnder thē, & last of all remaineth ꝑ 4.s. which the abiected pēs did

make, so

so I perceyue that I haue well doon. Nowe this wyll I not forget, but will this examination serue in all addition: Mayster. It serueth for all addition of sundry denominations, yf the addition be made with two lynes (as these were) elles it wyll not serue because that those summes, whyche are here added betweene the lynes in addition by one lyne, are vnderstande and not written, but I let that wayes passe, because as it is common, so is it more deceauable then this wates, namely if a mans memory be either dulle, other troubled. Scholar. Yette were good to knowe that waies also. M. If you desyre to knowe it, this it is in fewe wordes: Doo euery thynge as you dyd in this sorte, saue that where you made here two lynes, you shal make there but one: and those summes that you dydde here write betweene the lynes, you muste keepe in youre memorye, and adde theym (as you dyd here) eche one whan you come to his place. S. Than they differe not but in this, that this addition with two lynes leaueth nothyng to memory, but writeth downe all: and the other way committeth certayne numbres to memory, as you taught me in the fyrste examples of addition of smalle summes of one denomination. But what yf a man vse it (as you saye men doo commonly) how shall it be examyned? M. Seeyng you are so desyrous of it I will

I wyll shewe both an example of the addi-
tion, and also the manner to examyne yt, I
propose these .iij. summes to be added, and I gather fyrst the pens, as I dodde in the other sorte, and I finde of them 8,3,9, that is 20, of whych summe I bate awaye 12, which make 1 s̄, and keepe that 1 in my mynde, and the reste that is 8, I write vnder the pence. Than doo I adde the shyllinges togyther, and fynde of theym 6,7,8, that is 21, whereof I bate 20, that make 1 li, whiche I keepe in mynde, and to the other 1 that remayneth, I adde that one that came of the pence, and was in my mynde, whiche make 2, and them I wryte vnder the shyllynges.

li.	s̄.	d̄
12	8	9
6	7	3
3	6	8

Than doo I reckon the poundes togyther 3,6,12, that is 21, and to theym I adde the one in my minde, that remayned of the shyllynges, whych make 22, theym doo I wryte vnder the poundes, and than my summe totall appeareth to be 22, li. 2. s̄. 8. d̄. Now to examine this summe (and all alyke) you shall doo thus. Fyrst begin at the lefte hande with the poundes, and take from them that are aboue the lyne 9, as often as you can, than that, that remaineth, shall you double, than toyne it with the shyl-

lynges

lynges, and take away 9 frome that as often as you can, and what soeuer remayneth, you shal take for it 3 tymes so much and put to the pens, than take from all that summe 9 as often as you can, & what so remayneth after you haue withdrawē 9, as often as you can, write it at the end of a line, as I taught you in an other addition. And then come to the sum vnder the lyne begynnyng with the poundes, and doo euen as you did with the summes aboue the line, tyl you come to your pennyes, and if the figure of the summe that remayneth after taking away 9 (as often as you can) do agree with the other that remayned before of the other summe, which you dyd wryte at the ende of the lyne, than haue you doone well, els not. and for an example I wyll examine that last summe, whi che was thus.

	li.	s.	d
Fyrst I shall begyn at the left	12	9	9
hande with the poundes, put-	6	7	3
tyng them togyther, whyche	3	6	8
make 21, in whych summe I	22	23	8

fynde 9 twise, (for twyse 9 is 18) that I deduct, & there remaineth 3, that 3 must I double (as I sayde) bycause it is the remainer of the poundes, and it wyll bee 6. Than geether I the summe of the shyllyngs which is 21, to the whych I adde the forsayd 6, and than is it 27, wherin I fynd 9 thre tymes, and there remayneth nothing

This

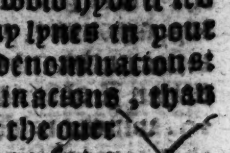
Thys remaynet shulde I take iii. tymes: but iii. tymes nothing, is nothyng: therefore in thys place is there nothyng left to be added to the pennies, therefore muste I take the summe of penies alone, which is 20, frome thens if I take 9 twyse, ther remaineth but 2, whych I put at the ende of a lyne, thus. 2——— Then I come to the poundes of the vnder numbre or totall summe, and ther I fynde 22, from whyche I take awaye 9 twyse, & there remayneth 4, that 4 I double, and it is 8, then do I adde that 8 to the shyllynges, and it maketh 10, from whyche I withdrawe 9, and there resteth 1, then doo I take that 1 thre tymes, and it maketh 3, whiche I adde to the 8 ꝺ, and it maketh 11, from whych if I take 9, there resteth 2, wyche is equall to the numbre noted at the ende of the lyne, & therby I perceaue that I haue done well. S. But I doo not see the reason of this. M. No, no more do you of many thinges els, but hereafter wyll I shew you the reasons of all Arithmetical operations, for this I iudg to be the best trade of teachyng, firste by somme briefe preceptes to instruete a learner somewhat in the vse of the arte, before he learne the reasons of the arte, and then may you afterwarde more soner make hym to perceaue the reasons: for harde it is for to occuppe a yonge learned witte wyth bothe the arte & the reasons of it all at ones

howe

how be it he shall neuer be cunnyng in decdt
man arte, that knoweth not the reason o
euery thyng touchyng it. S. Yet at the leas
I pray you shew me why did you write your
numbre that remaineth (after you had with
drawen al the nynes) at the ende of a lyne
for I sawe no reason why that lyne did ser
M. Dyd you euer marke a carpenter when
he wrought? S. Yea many tymes. M. And
haue you not sene him whan he hath taken
measure of a borde, that he hath prycked it,
and hathe wyth a twytch of hys hand dra
wen a line from the pryche that he made? S.
Yes I haue marked that, & haue seen some
make iii, or iiii, lines by the pryche, & some
also haue I seen make a crosse by it, but that
I perceiued was for ỹ easy fyndyng of the
pryche. M. And euen so is thys lyne, for the
easy fyndyng of your remayner, and ther
fore some doo make a crosse thus.
And set the one remayner aboue
the crosse, and the other vnder the
nether part of the crosse, as if I shoulde set
my two remayners thus.

But ther is an other sorte of profe
of addition, to which ỹ crosse ser
ueth more meeter. And that is when the ad
dicion is of dyuers denominations, and I
woulde examine euerye denomination by it
selfe, which wayes though it be not muche
vnlyke to the fyrste proue, that I brought of
suche

suche dyuers summes, yet I wyll declare it, least you should think that I wold hyde it frō you. You must make so many lynes in your crosse, as you haue sundrye denominations: as if you haue but 2 denominations, than you may make it thus, that the ouer parte and the nether part may serue for one denomination, and the two sides for the other. And yf you haue 3 denominations, as pounds, shillynges and pennies, than must you make three lynes thus. The vpright lyne maye serue for the pounds, and the highest thwart lyne for shillinges & the lowest for pēnes, as for example: I wyl take a summe thus added for the proofe, of the whiche, bicause it containeth thre denominations, I must make a crosse of 3 lynes thus.



li.	s.	d.
16	12	5
12	8	1
9	2	7
38	3	1

Than I recken fyrst at the right hand the pennies, 7, 1, 5 make 13, from whiche I take 12 for the nexte denomination, that is to say a shylling, and ther resteth 1 which I muste write at one ende of the nether thwarte line. Than I gather the summe of the shyllinges 2, 8, 12 which make 22, to theym I put one that I toke of the pennies, and that maketh

23, from those I take 20, the quantytye of the nexte greater denominacion, that is to say, a pounde, and there resteth 3, whych I write at the ende of the ouerthwart lyne. Than I adde togyther the poundes, 9, 12, 16, whiche make 37, to them I adde the 1 that came of the shyllynges, and than there is 38, wherein I synde 4 tymes 9, and 2 ouer, that two I write on the vpright lyne. Than I come to the totall summe, and examyne it, beginnyng at the pennies, where I synde but 1, and can not take 9 frome hym, therfore I set him at the other ende of the nether thwart lyne: than come I to the shyllynges, where I synd only 3, whiche bicause it is lesse than 9, I set it at the other ende of the line of shyllinges, that is the ouermost thwart lyne. Than of the 38 li. I take out 4 tymes 9 whych is 36, & there remayneth 2, whyche I wryte vnder the vprighte lyne. then I cõbyne euery numbre comparyng it to the numbre that is against it, and bicause I synd them to be euer one lyke his matche, I know that I haue wel don. S. This crosse I perceaue doothe serue for those thre denominacions, poundes, shyllinges, penies, but what is I had, ob q, quod, &c? M. You thinke you be at Oxforde styll, you bryng for the so taste your q and c. Those three lines, as I haue sayde, doth serue for thre denominacions, such as they be, as here they do serue for

poundes

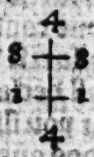
poũdes, shillynges, & pennies: but if you haue no pounde in your summe, than may they serue for shillynges, pennies, and halfpenies yea for q, q and c, if you haue no greater denomination, so that you remembre that the vpright line serueth for the greatest denomination, and the hyghest ouerthwart lyne for the next, and the lowest for the least. And so if you haue fewer denominations, you muste make your crosse with so many lines. And if that your summe be of more denominations make so many lines in your crosse. And thus will I make an end of Addition.

Examples of Addition.

li.	s.	d.	li.	s.	d.
362587		6	3492	17	10
41635		12	28	6	8
28124		2	13	13	4
4711		4	3821	17	10
33281	8	4			

The proofes.



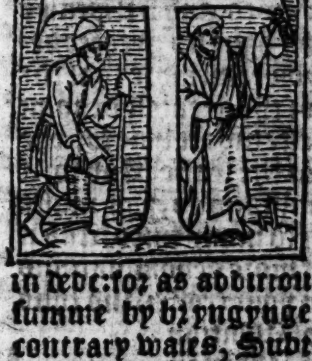


Another example.

li.	s.	d.	q.
2	0	2	
2 3	4		
10	2	3	
14	1	1	



Scholar.



When haue I learned the two fyrſt kyndes of Arithmetike, now (as I remembre) dooeth folowe Subtraction, whoſe name me thynketh doeth ſounde contrarye to Addition. M. So is it in dede: for as addition encreaſeth one groſſe ſumme by bryngynge manny into one, ſo contrary waies, Subtraction dymynyſheth a groſſe ſum, by withdrawing of other from it, ſo that Subtraction or rebatinge, is nothyng els, but an arte to withdrawe and abate one ſumme from an other, that the remainer may appear. S. What cal you the remainer? M. You may perceiue by the name. S. So me thinketh, but yet it is good to aſſure the trouthe of all ſuche thynges, leaſte in truſtyng to myn owne conjecture, I be deceiued. M. So is it the ſureſt waye. And as I ſee cauſe, I wyll ſtyll declare thynges vnto you ſo playnely, that you ſhall not need to doubte. How be it if I doo ouerpaſſe it ſometymes (as the manner of men is to forget the ſmal knowledge of theym to whom they ſpeake) than do you put me in remembraunce your ſelfe, and that waye is ſureſt. And as for this worde that you laſte asked me, take yee this

Deſcry-

description. The remayner is a summe left after dewe worckyng, which declareth the excesse or differeuce of the two other numbres: as yf I wolde deducte 14 oute of 18, there shoulde remayne 4, which is called the Remainer, & is the difference betwen those two numbres 14 & 18. S. I perceaue than what Subtraction is, now resteth to know the art to worke by it. M. That shall you do by this meanes: Fyrst you muste consydre, that yf you shoulde go aboute to rebate, you muste haue two sundry summes proposed, the fyrst whiche is youre grosse summe or summe totall, and it must be set byghest, and then the rebatement or summe to bee withdrawen, whyche muste be sette vnder the fyrst (whether it bee in one parcell or in manny) and that so, that the fyrst fygures be one iuste ouer an other, and so the seconde and thyrde, and all other folowynge, as you dyd in addition: then shall you drawe vnder theym a lyne, and so are your summes dewely set to begyn your worckyng. Then begyn you at the right hand (as you dyd in Addition) and withdraw the nether nũbre out of the byghher, and yf there remayne any thynge, write that ryght vnder them benethe the line: and yf there remayne nothyng (by reason that the two figures were equal) then write vnder them a cyphre of noughte. And so doo you with all the other fygures, euermore abatyng

Remai

tyng the nether, out of the higher and write vnder them the remainer styll, tyll you come to the ende. And so wyll there appere vnder the lyne, what remayneth of youre grosse summe, after you haue deducted the other summe from it, as in this example: I receyued of your father 48 ℔, of whiche I haue layd out for you 36 ℔, nowe wolde I know what dothe remayne, and therfore I set my noumbres thus in order: Fyrste I write the greatest summe, and vnder hym the lesser, so that the figures at the right side be euen one vnder an other, and so the other thus.

℔.
48
36

Than do I rebate 6 out of 8, & there resteth 2, whiche I write vnder theym ryght beneth the line thus.

℔.
48
36
2

Than I go to the seconde figures, and doo rebate 3 out of 4, where there remaineth 1, whyche I write vnder them ryght, and then the holle summe and operation appereth thus.

℔.
48
36
12

Whereby it appereth, that if I withdraw 36 out of 48, there remaineth 12. ℔. Now wyll I proue in a greater summe: And I wylle subtracte 2367924 out of 3468946. Those summes I set in order thus.

3468946
2367924

Then doo I begyn at the ryght syde, and deducte 4 out of 6, & there resteth 2, whyche I write vnder them. Than go I to the seconde fyg-

ure

res, and withdrawe 2 out of 4, and there remayne 2, whyche I set vnder theym also: than I take 9 out of 9, and there resteth 0, which I write vnder them: for you say, that if the figures be equall, so that nothyng remayne, I muste write this cyphre 0 vnder theim. M. It was well remembred now go forth. S. Then come I to the fourth place and draw 7 out of 8, and there remaineth 1 whyche I write vnder theym also; than in the v. place I take 6 frome 6, and there resteth nought, for it I write vnder them a cyphre, 0: than in the vi. place 3 rebated from 4, remayneth 1, which I write vnder theme and likewise, in the. vii. and laste place, 2 taken frome 3, there is lefte 1, which I write vnder them: so haue I done my whole worke, and my summes appeare thus.

34689462	whereby I se, that if I rebate
23679242	23679242 out of 34689462
11010220	ther remaineth 11010220. M.

This is wel done And that you may be sure to perceaue fully the art of Subtractiō, let me se how can you subtracte 529847322 out of 8250003456. S. First I set downe the greatest summe, and then I write vnder it the lesser numbre, beginnyng at the right syde, and then my figures will stande thus.

	8250003456
Than take I 2 frōm 6, &	529847322
frest is 4, which I write	

vnder them, then doo I withdrawe 3 from 3, and there remayne 2, whiche I wryte vnder them: then take I 7 out of 5ower, but that I can not, what shall I nowe doo? M. Marke well what I shall tell you nowe, how you shall do in this case and in al other like. If any figure of the nether summe be greater then the fygure of the summe that is ouer hym, so that it can not be taken out of the figure ouer hym, then must you put 10 to the ouer fygure, and then consider how much it is, and oute of that whole summe withdrawe the nether figure, and wryte the relle vnder them. Canne you remembre this? S. Yes that I trust I shall. Nowe then in my example where I should haue taken 7 out of 4 and could not, I put 10 to that 4, whiche maketh 14, from it I take away 7, and there resteth 7 also, whyche I wryte vnder them. M. So haue ye doone well, but nowe must you marke an other thynge also: that when so euer you doo so put 10 to any figure of the ouer numbre, you must adde one fyll to the fygure or place that folowyth next in the nether lyne, as in this example there folowyth 4, to whiche you muste putte 1 and make hym 5, & then goon as I haue taught you. S. Then shall I say 4 and 1 (whiche I must put to hym for the 10 that I added to 4 before) make 5, which I should take out of 3, but that can not be, therfore muste I

put

put to it also 10, and then it wil be 13, frome whiche I take 5, & there resteth 8 to be writ ten vnder them: and because of that 10 added to the 3, I muste adde 1 to 9 that folo weth in the nether lyne, and maketh 9, whi che I shuld take out of 0 and can not: ther fore I put therto 10 & that maketh 10, frome 10 I take 9, & there remaineth 1, whiche I write vnder the. Then do I adde 1 likewise to the nexte figure beneth which is 9, and it maketh 10, that 10 should I take out of the figure aboue, but I can not, for it is 0. ther fore I put 10 to it, and so take I 10 out of 10, and there resteth 0 to bee writ ten vnder them: then come I to the nexte fygure why che is 2, and to hym doo I adde 1, whyche maketh 3. & 3 I can not take out of nought, therfore of that naught I make 10, & thenne doo I take 3. so remaineth there 7 to bee written vnder them: lykewyse doo I take 1 to 5 that foloweth, & then is it 6, that wolde I take out of 5, and can not, therfore I take 10 to that 5, and make it 15, frome whyche I rebate 6, and there remayneth 9, whyche I write vnder them: now haue I spente all the nether figures, & what shall I do more? M. You shulde haue added 1 to the nexte fi gure folowyng (yf there hadde ben any) be cause you added 10 to the last fygure before of the ouer lyne. But seeyng there is no fygure folowyng, you muste adde that 1 to

the place folowyng, and then deduct that 1 from the numbre aboue. S. Then shalt I say because I borowed 10 to the ouer 5, I must put 1 in the next place beneth, that is vnder 2, then muste I subtracte that 1 frome 2, and there resteth 1 to be writen vnder that 2 in the nyneth place: now I haue no more to subtracte, for there is neuer a figure remaining beneth, nother yet any vnitee to be added, by reaule I borowed not 10 to the figure laste before, and yet is there 8 remaynyng in the ouer lyne, whiche (I thynke by reason) shoulde be set at thende of the lygures in the lowest rowe, whyche is vnder the lyne, for because there was nothyng taken from it. M. That is well considered, and reason teacheth so in dede: howbeit (as I sayde before) I wyll omitte the reasons, tyll another more conuenient tyme, and wyll only at this tyme teache you the practyle of the art, that you may exercise youre selfe some what in it, in the mean tyme, by proper and witty questions, as I wyll teache you some before we departe. And when you haue well occupied your mynde, and quickened youre wytte in in it, than wyl I afterward at youre retourne, instructe you in the reasons of all this woorkyng, that then you maye woorke perfectly, when you see the reasons in euery thyng, why you shuld so doo. S. Syr, well I may pray to God to recompence this youre

good

goodnes, but surely I can not do it with any temporall benefite, neither any other man in my behalfe. for what treasure is there to be compared to the riches and treasure of learnyng? But syr (I beseeche you) shall I always, when any numbre so remaineth alone (as this 8 dydde) write hym vnder the lyne straight against his owne place? May. Yea what els, whether they be one or many, and this well remembred, you haue learned subtraction. Howbeit, because of certayn thinges that myght deceaue you, if you dyd not take good hede to your worknyng, I will propose to you an other exãple of many nũbres to be subtracted, as thus. I receaued of a frende of myne to keepe 2869 crownes, of which at one tyme I deliuered hym agayne 500, at an other tyme 368, and at an other tyme 440, and an other tyme 80, and an other tyme 64, nowe wold I knowe howe many doth reste behynde. Therfore first I set down my grosse sũme, & vnderneth hym I set all the parcels, thus, & vnder theim a double line. Then syrste I begynne at the first place, and gather togither the summe of all thole lynes (saue the ouermoste) in theyr syrst figures, and so do I with all the sygures of the seconde place, and so forthe, as I dyd in addition.

2869	
500	
368	
440	
80	
64	

saue

saue that I leaue out the highest row of nũ-
bres: and that summe so gathered betwene
the lynes, doo I subtract out of the highest
row of numbres, and the remaynder doo I
set vnder the vtthermost lyne, as for exãple.

I set the summes as before:	2 8 6 9
than doo I gather the fyrste	5 0 0
figures togyther, where I	3 6 8
fynde but 4 ♯ 8 that make	4 4 0
12, for thre ciphres encrease	8 0
no summe in Addicion, as	6 4
you learned beefore of the	1 4 5 2
12, therfore do I wryte the	1 4 1 7

digite 2, and kepe the article in my mynde,
tyll I come to the seconde places, where I
fynde 6, 8, 4, 6, that make 24, to theym I
put the article in my mynde, and it is 25, of
whiche I wryte 5 vnder the seconde place,
and keepe the digite 2 in my mynde for the
iij. place, where I fynde 4, 3, 5, that make
12, to the which I adde the 2 in my mynd,
and that maketh 14, therof I wryte the 4,
vnder the thyrde place: and bycause there re-
mayneth no more figures to bee added, I
wryte the digite 1 in the fourth place, as you
see in the exaumple. Than come I to subtra-
ctynge of this summe, and I shall subtract
the summe betwene the lines, from the ouer
most summe, saying: 2 from 9 remayne 7,
to be wryten vnder them, beneth the lowest

lyne,

ine: than in the ſecond place I take 5 from 6, and there reſteth 1 to bee wrytten vnder them. Than in the thyrde place, 4 from 8 reſteth 4. In the.iiij. place 1 from 2, remayneth 1. And thus I ſee, that after thoſe 5 ſummes are ſubtracted from 2869, the remainer is 1417. Scho. This I perceaue: but is there no ſhorter way? more ſpeedier? Maiſt. Yes, when you are a while exercyſed in it: for you may as faſt as you can, gather the numbres togyther, withdraw them out of the higheſt ſumme, if ſo be it, that all the parcelles whych you do gather, doo not exceed nyne, but and yf they exceede nyne, than muſte you ſubtracte onely the digete that is in it, and reſerue the article tyll the nexte place, where you ſhall adde it with the other figures, and ſo ſubtract the whole out of the figure aboue them: but and if in this place the ſumme of the parcels do exceede 9, than (as I ſayd before) ſubtracte the digite onely, & reſerue the article to the next place, and ſo tyl go forth, tyl you haue ended your worke: as for example in the laſt ſummes propoſed, I gather tyll in the fyrſt place 4, and 8 that maketh 12, of whyche I deducte the digite 2 oute of 9, and write vnder the remaynder, whyche is 7, and the article 1, I kepe in my mynde. Than in the ſecond places I gather the parcels, 6, 8, 4, 6, whyche amount to 24, to that I adde the article 1,

whych

whyche I haue in my mynde, and than is it 27, than doo I take 5 (that is the diget in this numbre) from 6, that is in the seconde place of the highest summe, and ther remaineth but 1 to bee wryten vnder theym, and nowe doo I keepe the article 2 in my mynde styll. Than in the thyrde place, 4, 3, 5, maketh 12, and the article 2 in my mynde maketh 14. Whē take I 4, (which is the diget) from 8, that is ouer them, and three resteth 4, which I wryte vnder theym. Then haue I the article one yet in my mynde, whych I shuld adde to the parcels next folowyng, but seyng there is no numbre folowing, I take that diget alone, and deduct him out of the next summe aboue, whyche is 2, and than is the remainer 1, which I wryte in ỹ fourth place vnder 2. Lo, nowe haue you a shorter waie. S. I lyke both wayes well and I perceaue bothe well, yet as in the one, the woorkyng seemeth somewhat long, so in the other it leaueth verye muche (me seemeth) to remembraunce, and therfore may cause erroniously, excepte a man haue a quycke & an exercised remembraunce. M. What, would you then haue suche a waye that should not be so longe as the one, nor so shorte as the other? S. Yea if there were any suche. Mayst: Than do thus: styll as you gather your parcels when they exceede a diget, and maketh 10 or more, take the article, and wryte hym

be

betwene 2 lynes (as in ꝑ firſt exãple) vnder the nexte place towarde the left hand, and the deduct the diget from the figure that is ouer him, and wꝛite the remainer. And the wꝛen you gather the next parcels, you ſhall ad to them the figure that is vnder them, betwene the 2. lines and if it excede 9, do as I ſaid before, wꝛite the article vnder the next place betwene the lines, & ſubtract the diget frõ the figure that is ouer thoſe parcels: & if that all the parcels together & the numbꝛe betwene the lines do make but a diget, then deduct it wholy from the fygure abouet as in thys example. I wolde ſubtracte 40308964 out of 40308964, theſe 2 parcelles 20003428. illi. 10002342. 10002342. 10101461, 10101461, therefore I ſet them fyꝛſt in oꝛder bewe, and then I gathꝛ the parcels of the fyꝛſt place, whyche are 8, 2, 1, that is 11, of whyche I take away the article, and ſet hym vnder the ſecond place betwene the lines, & the diget 1 ꝛemayneth, I deduct out of 4, and there reſteth 3 to bee wꝛytten vnder the fyꝛſte place beneth the lowe ſtyne. Then come I to the ſeconde place, and gather the parcelles of it 6, 4, 2, & the 1 betwene ꝑ lines, which make 13, of whych I take the article and ſet him vnder the thirde place, betweene the lynes, and the dyget 3 I take from 6, and there re-

mayn-

maineth 3, which I write vnder the second place benethe the lowest lyne. Than in the thyrde place I fynde 4, 3, 4, whyche wyth the 1 betweene the lynes, doe make 11, therfore I write the article again vnder the fourth place, and the digite 2 I take from 9, and there remayneth 7, 2, whyche I wryte vnder theim beneth the lowest line. And thanne come I to the fowerth place, where I gather 1, 2, 3, and the 1 betweene the lynes, that maketh 7, whiche bycause it is but a digite, I pluck from 8, and the remayner is 1, and must be wrytten vnder them in the fowerth place. Than come I to the fyfth place, where are only thre cyphers, whyche make nothing, thā shuld I take that, that is to say, nothyng, from the figure ouer them, whiche is also a cyphar, therfore I must say thus: yf I take nought from nought, there remayneth nought, so must I write a cypher vnder them. Than in the vi. place I fynde but 1, whyche I take out of 3 ouer him, and the remayner is 2, that must be wrytten beneth the lowest lyne in the vi. place. So go I to the seuenth place, wher I find only cyphers, and in the grosse summe ouer them a cypher also, therefore must I wryte theim in maner (whiche is nothynge) wyth a cypher also. Than in the viii. and laste place, I gather 1, 1, 2, that make 4, whych if I take out of that foure that is ouer them, there wyl no

thyng remayn, and that must be noted with a cypher benethe the lowest lyne, as I haue often sayd. And thus haue I ended my worke and the figures stande thus.

Scho. Syr, I remembre you taught me that cyphres shuld not come in the last place, forbicause they serue onely to increse the valew of other figures whyche folowe them, and serue not for those figures that go before them: and nowe in your example you haue set two cyphres in the two laste places Mai. I cõmend you for your remembrance, and truth it is, I shuld not haue sette theym here, but onely bycause that I wolde make you playnly to perceiue the art of subtraction, therfore seyng you doo nowe perceaue it, whan so euer you shuld write down a cyphre loke whether any other figures be yet behid, and if not, than let go the cyphre also, for it nedeth not to write him in any latter places wher no other figure doth folow, except it be (as I did) to teach the vse of subtraction the plainer, therefore my figures must stad thus, whã I haue ended my woorke. S. So wold I thinke by that you taught me before. And now I thinke I could sub

4	0	3	0	8	9	6	4	
2	0	0	0	3	4	2	8	}
1	0	0	0	2	3	4	2	c
1	0	1	0	1	4	6	1	c
			1	1	1			}
0	0	2	0	1	7	3	3	

4	0	3	0	8	9	6	4	
2	0	0	0	3	4	2	8	
1	0	0	0	2	3	4	2	c
1	0	1	0	1	4	6	1	c
			1	1	1			
	2	0	1	7	3	3		

tract any lyke summes. M. So may you, yf you haue marked what I haue taught you. But bycause this thyng (as all other) muste be lerned surely by often practise, I wyl propounde here. ij. examples to you. whiche yf you often doo practise, you shall be expe and perfect to subtract any other summe lightly, for in it is contayned all the obseruances of whole nūbre. And because you shal perceaue somewhat both how to do it, & also whether it be wel doon, when you haue proued to do, therfore haue I written vnder theim both their remaynders.

308964		125614	
103145		342	
102597		681	
101024		201	
11		11	
2198		124390	

Schol. Syr I thanke you, but I thynke I myght the better doo it, if you did shewme the woorkynge of it. M. Yea but you muste proue your selfe to do some thynges that you were neuer taught, or els you shall not be able to doo any more then you were taught, it were rather to learne by rote (as they call it) then by reasō: And agayn there is nothyng in this exāple or any other of whole nūbre, but I haue taught you the rules of them all ready. S. Then I trust by practise to attain the vse of it. And is this al that I shal lerne

of

of Subtraction? M. Yea, sauyng that (as you haue sene in Addition) there are broken numbres or fractiõs, in which the worky̓ng is not muche vnlike, yet without some instructions be geuen of it, it myght seme to a lerner more difficult than in dede it is, therfore I will breefely shew you the vse of it onely by one exãple or. ij. A certayn man owed to me 14 li. 12 s, 8 d, of whiche he payd me at one tyme 4 li, 6 s, 8 d, at an other tyme 3 li. & at an other tyme 2 li, 3 s 4 d, and laste of all 6 s. 8 d, now wold I know what remaineth vnpayde yet: Therfore I set my summes thus.

	li.	s.	d
14	12	8	
4	6	8	
3			
2	3	4	
	6	8	

S. Why I pray you, why do you write 2. li? for the comon spech vseth rather to say 40 s. M. We must here vse the denomination that is greatest in any summe, so that we may not write, accordyng as we vse to speake, saying 16, d, 18 d, or like wais 7 grotes, 8 grotes, 24 s. 40 s, 48 s. and such other, but we must write euery denomination that is in any summe by it self, namely shyllynges & poundes, so muste wee write for these summes now named 1 s. 4 d, 1 s. 6 d, 2 s. 4 d, 2 s. 8 d, 1 li. 4 s. 2 li. 8 s. and so foorthe of other lyke. S. So that we may not write in Arithmetike penies, whan the summe amounteth to shyllynges, nor shyllynges whan the summe maketh poundes.

Nowe(t it pleaſe you, ende your exaumple.

M. Whan my ſummes are ſo ſette as I ſhewed, than muſt I begynne with the ſmalleſt denomination, ſaieng, 8, 4, 8, are 20, whiche ſumme becauſe it is pens, and 12 peus dooe make 1 ſ, I muſte take frome that 20, whiche commeth of the iii. parcels 12, & for theim wryte 1, betweene the lynes vnder the ſhillynges: than the 8 that remayneth, I take oute of the hygher ſumme (whiche is 8 alſo) and than remayneth naughte, wherfore vnder the peuies I wryte nothyng. Then come I to the ſhillynges, & gather the parcels 6, 3, 6, which with the 1 betweene the lynes make 16, that muſt I take out of the ſumme that is ouer it: but ſeyng that ſumme is but 12, I can not take 16 out of 12, I muſt borowe 1 of the 14 li, & put to the 12, & that maketh 32, for 1 li. is worth 20 ſ, than take I 16 out of 32, and there reſteth 16, to be wryten vnder the ſhyllynges. Than come to the poundes, whole parcels are 2, 3 4, that is in all 9, & 1 more muſt I adde therto, becauſe of the 1 that I borowed before vnto the 12 s. and than is there 10, whyche I muſte take oute of 14, ſo doothe there remayn 4, to be wryten vnder the poundes: ſo doth my remayner appere

	li.	ſ.	d.	
	14	12	8	9
	4	6	8	
	3			
	2	3	4	
		6	8	
		1		
	4	16		

pere to be 4 li, 16 ß S. This doo I perceaue very well, and yt there be none other thyng to bee learned in subtraction, than maye I com to multiplication, for that you reknedrert. M. We haue don in dede wyth the arte of subtraction, as touchyng the worckynge. But yet before we go to multiplicatiõ, I wil instruct you howe to examine your woorke whether it be wel doone or no, and that is by castynge away 9 as often as you can fynde it (as you dyd in Addition) sayyng that you must here examin the highest numbre alone and note the residue of it at a lynes end, as you did in addition, and when you haue don with the hyghest numbre, than examine all the other togyther, castyng thens 9 as often as you can: & if the remainer be like thother, than haue you doon wel. But if you haue diuers denominations in your summe, yet for them all shal you make but one seuerall line as you dyd in Addition, remembryng to begyn the examinatiõ at the greatest denominacion, & to double the remayner of poũds, and triple the remainer of shyllyngs, as you dyd also in Addition. As for

	li.	ß.	d.
a proofe, I wil examine thys	14	12	8
woorke, wherin the highest	4	6	8
line I fynde of poundes 14,	3		
from thens I bate 9, and	2	3	4
there resteth 5, whych I do		6	8
double, bicause thei ar poun-	4	16	

des

des, and than ar they 10, therto I adde the 12, and it maketh 22, from whyche I take 9 twise, & there resteth 4, whiche bicause they ar shillynges, I triple, & then ar they 12, therto I adde the 8, and than are they 20. thense take I twise 9, and yet resteth 2, whiche I wryte at the one ende of a lyne thus, 2——

Than I examine all the other parcels and the remayner togyther, euerye denominatiō by itselfe. And fyrst of pounds I fynde 4, 3, 2, 4, that is 13, from whych I take 9, and there resteth 4, that do I double, and it maketh 8, to it do I put the shyllynges, 6, 3, 6, 16, that is 31, for the one betweene the lines must not be rechned, nor none in that space, and that maketh in all 39, where hense I take 9 four tymes, and there remayneth 3, that do I take iij. tymes, & it is 9, wherefore I call it away: than do I take the pences, 8, 4, 8, that make 20, from whiche I take 9, twyse, and there resteth 2, whych I wryte at the other ende of the proofe lyne, and bicause I see that those ii. numbres ar equal, I saye that I haue well wrought. And if you wyll you maye make for euerye denomination a line as you learned in addition: but thā must you begin your examination at the smalest denomination, as you dyd in Addition, for their proofe is al togyther like, saving that in addicion you examined the nethermost summe alone, & al the other togyther, & in subtractiō

you

you must examine the highest numbre alone & all the other togʒther. And if you marke it well, it is euen alone, foʒ that summe that in Addition is lowest, in subtraction is highest and that summe is called the grosse oʒ totall summe. Therfoʒe if you marke what I said in addition, you may easyly perceiue what is to be don foʒ the proof of subtraction: And to thentent ỷ you may perceaue it the better, I wyl sheʷe you an other proue of subtraction & that shal be by addytiō, thus. Dʒaw vnder the lowest nūbʒe, which is your remayner, a lyne, thā adde that nūbʒe and all the other that you did subtract before, togʒther, & wʒite that that amounteth, vnder the lowest lyne, & if the summe that cometh therof be equal to the highest at the subtraction, than is the subtraction wel wʒought, els not: as foʒ example in the last summes, which stode thus

Grosse totall summes.

li.	s.	d.	
14	12	8	
4	6	8	
3			
2	3	4	
	6	8	
1	1		
4	16		

First I adde 8, 4, 8, that maketh 20, whereof I take 12 awaỷ (bycause they make one shylling) & wʒite foʒ the 1 vnder ỷ shillings, & the 8 ỷ is left, I wʒyte beneth ỷ lowest line, thā adde I the shylynges, 6, 3, 6, 1, 16, ỷ make 32, frō whiche I take 20, and foʒ it I wʒite one vnder the pou̇des, & the 12 that remaineth, I wʒyte vnder the shylynges. Than come I to the pounds.

addyng them togyther, whych ar 4,3,2,1,4 that maketh 14, than do I wryte 14 vnder the li, and so haue I ended the addition, and I se that the lowest lyne of numbre, and the highest be lyke, wherfore I know ꝑ I haue wel done, for my figures appere thus.

li	ſ.	d.
24	12	8
4	6	8
3		
2	3	4
	4	8
1	1	-
4	16	
14	12	8

And thus nowe haue I taught you the art of Subtraction, and the meanes to proue whether it bee welle wrought or not. Now and you remembre, I omitted in rechynge the proofe of Addition one waye, whych I sayde was by Subtraction. S. Truth it is, and than was it deferred, bicause ꝑ I had not than learned the feat of subtraction, whereby I shuld haue proued it: but nowe I thanke you, I haue wel lerned the art of subtractiõ & the proues of it, both by 9 and by addition. And now I wold be glad to know how I may proue Addition by subtraction. M. Than marke you this. whẽ you haue ended your additiõ. take the nũbres al that you did adde to the highest summe, & deduct or subtract them frõ the grosse sum that doth resolue it, & if the remaunt be like to the highest nũbre, thẽ haue you done wel, els not: as for exaple I take one of the sũmes ꝑ I did adde before, which was this,

The proofe of Subtraction.

Than

Then doo I come to the myddle numbre, and subtract that from the nether numbre, begynnyng at the lefte hande, and fyrste I say, o out of o, there remayneth o that write I vnder an other lyne. Then agayne o (in the seconde place) from o remayneth o, vnder it I write o also. Then in the thyrd place 4 out of 2 wyll not bee, therefore I adde to that 2, 10, and make it 12, from that I take 4, and there resteth 8. Then saye I farther. 9 in the iiii. place, 1 (whyche I muste adde for the 10 borowed beefore) make 10, that must I take from 6, and because I can not, I adde to the 6, 10, and then is it 16, from thence I take 10, and there resteth 6 to bee written vnder them. Then in the v. place where I fynd nothyng wrytten, I must set 1 for the 10 laste borowed, and that I do I take frome the 1 vnder hym, & so remaine in naught, wherfore I write doune a cipher o. Now haue I doone wyth the subtraction, & yet in the grosse summe remayneth 1, which I must set ryght in the same place, in the remainer and in the remainer appeareth to be lyke vnto the hyghest summe of the addition, as here appereth, wherfore I saye that the Addition was well wrought, and thus may you do in any other summe of anye denomination of many. Wherfore now wyl

1 0 6 8 0 0	
9 4 0 0	
1 1 6 2 0 0	
1 0 6 8 0 0	
9 4 0 0	
1 1 6 2 0 0	

I make an ende of Subtraction, and wilbe
ructe you in Multiplication.

Multiplication.



Multiplycation is an operation by.ii. summes prodoryngt the thyrde, whych so many tymes shall contayne the fyrste, as there are vnities in the seconde. And ye ferueth in the sted of many additions. as for example. If I wold know how many are thyrty tymes 48. If I shulde adde 48, thyrty tymes, it wouldbe a longe worke, therfore was thys worke of Multiplication diuyded, whych shal do that at ones, that Addition should doo at many tymes. S. I perceaue the commoditie of it, partely, but I shall not see the full profyt of it, tyll I knowe the whole vse of it, Therfore fyr, I beseke you teache me the worckynge of it. M. So I iudge it beste, but bycause that great summes can not be multiplied but by the multiplication of digettes, therefore I thynke beste, fothewe you fyrste the arte of multiplying them, as when I say. 8 tymes 8, or 8 tymes 9, &c. And as for the small dyggetes vnder 5, it were but folly to teach any rule, seyng they are so easy, y euery child can

Multiplica-
tion of dy-
gits.

doo it: but for the multiplication of the grea
ter digets, thus shal you doo. Fyrst set your
digettes one ouer the other right, than loke
how many eche of them lacketh of 10, and
wryte that agaynst eche of them, and that is
called the differences: as yf I would know
howe many are 7 tymes 8, I muste wryte
thyse digettes thus.

The differences.

8
7

Then doo I looke howe much 8 doth
differe from 10, and I fynde it to be 2,
that 2 doo I wryte at the righte hande of 8,
thus.

8 2
7

Then doo I take the difference of 7,
lykewayse from 10, that is 3, and I
wryte that at the righte syde of 7, as
you see in this example.

8 2
7 3

Than do I drawe a line vnder
them, as in Addition, thus.

Then doo I multiply the two differences, say
yng, 2 tymes 3 make 6, that muste I euer
set vnder the differences beneth the lyne: thã
muste I take the one of the differences, (why
ehe I wyll, for all is lyke) from the other di-
get, not from his owne, and that that
is lefte, muste I wryte vnder the dy-
gettes, as in this example.

8 2
7 3
5 6

If I take 2 from 7, or 3 from 8, there re-
maineth 5, that 5 muste I wryte vnder the di
gettes, & than ther appereth the multiplica-
tion of 7 tymes 8, to be 56, & lyke of anye
other digets, yf they be aboue 5, for yf they

bee

be vnder 5. then wyll their differences be gre-
ter than them selfe, so that they can not be
taken out of theym. And agayne suche lyt-
tle summes euerye chylde can multyplye, as
to saye, 2 tymes 3, or 4 tymes 5, and suche
lyke. S. Truth it is, and seyng me seemeth
that I vnderstande the multiplying of the
greater dygets, I wyll proove by an exam-
ple how I canne doo it. I would know how
many are 9 tymes 6. M. It is all onein ma-
ner to saye 9 tymes 6, or 6 tymes 9. but yet
the order is beste to put the lesser fyrste
saying 6 tymes 9, and so of all other sum-
mes. S. Then wold I know how many are
6 tymes 9, therefore I set the diget- 9
tes thus. 6

Than doo I sette their differences at their
right syde, the dyfference of 9 whyche is one
agaynst it, & the differeence of 6, whych is 4
agaynste it also, as in this example.

9 1 And vnder theym I drawe a lyne,
6 4 than doo I multyply the digettes to-
gyther, saying 1 tyme 4 maketh 4,
that 4 do I wryte vnder the differences, thus.
Then take I one of the differences 9
from the other diget, as 1 frome 6, 6
or els 4 from 9, & the ways there 4
remayneth 5, whyche I doo wryte vnder the dy-
gettes, and so appeareth the multiplicatio
of 6 tymes 9, to be 54. Thus I se the lear-
of this maner of multiplicatiō of digets. N

Now

Nowe myght you go straight to the Multiplication of greater nũbres, saue that both for youre ease and surette in workynge, I wyll drawe you here a table, whereby shall appere the multiplication of all digets, and this is it.

A table to multiply all digettes by.

1	1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18	
3	9	12	15	18	21	24	27		
4	16	20	24	28	32	36			
5	25	30	35	40	45				
6	36	42	48	54					
7	49	56	63						
8	64	72							
9	81								

In whiche figure whan you woulde knowe any multiplication of digets, seke your first or taste diget in the black squares, and from it go ryght forth towarde the ryght hande tyll you come vnder the digete of youre seconde digite, whiche is in the hyghest rowe, and then the numbre that is in the metynge of their both squares, is the multiplication that amounteth of theym. As yf you wolde knowe by this table the multiplication of 7 tymes 9, seeke fyrst 7 in the blacke squares, and than go ryght forth towarde the right hand

hande, tyll you come vnder 9 of the highest rowe, & in the metyng of their squares, you may se 63, which is their multiplication. S. This is very good and ready, and so maye I fynde the multiplication of any digettes. But now how shal I do in greater summes?

To multiply greater summes.

M. When you would multiply any summe by an other, you shall marke that it is the meetest order to set the greatest numbre highest, whiche is the place of the noumbre that must be multiplied: and lyke wayes the lesser numbre vnder it, for that is the place of the multiplier or multiplicatour, that is to say, the numbre, by which multiplication is made: and is in englyshe alwaies put before this worde tymes. in suche speakyng, when I saie 20 tymes 70, and the noumbre that folowethe this worde, tymes, is that whych must be multiplied. Therfore when I wold multiply one numbre by an other, I muste write the greatest byggest, and the lesser vnder it, as in Addition. And vnder them must I drawe a lyne, as for example: If I woulde multiplye 264 by 29, I must sette theym thus:

Multiplier

Tymes.

264
29

Than must I multiply euery figure of the hygher row, by euery figure of the nether rowe, and that that amounteth I must set vnder the lyne, as thus: fyrst I do multiply 4 by 9, sayeng, 9 times 4 (or 4 times 9 whiche is all one) & that maketh 36, as the table

table before of digettes doth declare, of that 36 I must write the 6 that is the diget. vnder the 9, and the 3 in the next place towarde the lefte hande.

Then come I to the seconde figure of the higher rowe, & say, 9 tymes 6, make 54, of whyche I write the 4 vnder the 3, and the 5 vnder the next place (as the reason wylleth me) thus.

```
264
 29
536
  4
```

Then come I to the nexte fygure, which is 2, and do multiply it by 9, & that maketh 18, wherof I wryte 8 vnder the thyrde place, and the article 1 in the fourth place, thus.

```
264
 29
1536
 84
```

And then haue I ended the fyrst figure of the multiplier. Then begin I with the nexte figure, and multiplye it into all the hygher fygures, as thus.

```
264
 29
1536
 84
  8
```

Fyrst 2 tymes 4, make 8, that do I write vnder ỹ secõde place, for euermore the diget or first figure of the multiplication ỹ amounteth of the first figure of the higher numbre, must be set vnder the multiplier of it, and the other in their ordre toward the left hande. S. I vnderstande you thus, that the diget of the summe amountynge of the multiplicatiõ of the first figure of the higher rowe, by the firste figure of the lower row or multiplier, must be set vnder the first place:

and

and that that amounteth of the same fyrste figure by the seconde multiplier, muste bee set vnder the seconde place: and so of the other, if there be more multipliers. M. So meane I in bede: and if there amount but a figette, than must it be set vnder the multiplier. And nowe to go foorth, I multiply by the same 2, the seconde figure of the higher rowe, whiche is 6, saying: 2 tymes 6, make 12, wherof I write 2 diget 2 vnder the third place, and the article 1 I write vnder the fourth place.

Then doo I multiply the laste figure of the higher sume, by that same 2, saying: 2 tymes 2 is 4, whiche I write vnder the fourthe place, and so haue I ended the hole multiplication, and the summes stande thus.

Than muste I drawe a lyne vnder all those summes that amount of the multiplication, and muste adde all theim into one summe, as in this exaumple you maye see, where in the fyrste place I fynde but 6, and therfore write I it vnder the lyne: then in the seconde place 8, 4, 3, make 15, whereof I write 5, and keepe 1 in my mynde, and so foorthe as you learned in Addition, and so apereth the hole summe

	264
	29
	1536
	184
	28
264	
29	
1536	
184	
428	
7656	

summe to be 7656. whyche amounteth of the multiplication of 264 by 29. Schol. If ther be no more to be obserued in it, the can I doo it I suppose, as by this exaumple I shall proue. I would multiply 1365 by 236, wherefore I set them thus.

Then doo I multiplye 5 by 6 saying, 6 times 5 make 30, of whiche I write the cypher in the fyrst place, and the article 3 in the seconde place

Than doo I by the same 6 multiply the second fygure of ẏ bygher summe, which is 6, saying: 6 tymes 6 make 36, of whyche I write the 6 vnder the second place, and the 3 vnder the thyrd place.

Than doo I multiplye the thyrde figure, whyche is 3, by the same 6, and that maketh 18, of that I set the 8 vnder the thyrd place, and the one in the fourth place.

Than come I to the laste figure of the bygher summe, and multyplye it by 6, sayinge 6 tymes 1 make 6, that doo I wryte vnder the fourth place.

And so haue I ended the fyrste multiplier, than begynne I with the seconde multiplier, and saye fyrste 3 tymes 5, that maketh 15, of whyche I set the 5 vnder the second place bicause that the multiplier is ther, & the 1 I

1365	
236	
30	
1365	
236	
330	
6	
1365	
236	
1330	
86	
1365	
236	
1330	
686	

ſet vnder the thyrde place. Than com I to the ſecõd figure that is 6, and multiplye it by 3, whych maketh 18, of whych I ſet the 8 vnder the thyrd place, and the article 1 in the fourth place.

Than come I to the thyrde figure which is 3, and multiplye it by 3, ſayng: 3 tymes 3 make 9, whych becauſe it is but one bygette, I ſet vnder the fourthe place.

And then commynge to the laſt fygure 1, I multiply it by 3, and it maketh 3, whiche I ſet in the fyfte place, and than haue I ended if. of the multipliers, & the ſummes ſtande thus.

Then come I to the thyrde multiplyer, & multiplye it into euerye fygure of the bygher ſumme. and firſt I ſaye. 2 times 5 make 10, of whyche I ſette the cypher vnder the multiplier in the thyrde place, and the article 1 in the fourthe place.

And ſo multiplyng the ſecond fygure 6, by that ſame 2, there amounteth 12, wherof I write the digitte 2 vnder the fourth place, & ỹ article 1 vnder ỹ v. place. ¶ Now do I multiplye by the ſame 2 the thirð figure of the bygher ſumme

1	3	6	5
		2	3 6
1	3	3	0
6	8	6	
1	1	5	
9	8		
1	0		

1	3	6	5
		2	3 6
1	3	3	0
6	8	6	
1	1	5	
9	8		
1	0		

1	3	6	5
		2	3 6
1	3	3	0
6	8	6	
1	1	5	
9	8		
1	0		
	2		

which

whych is 3, and that maketh 6, whyche I
set vnder the fyrst place. Than come I to the
laste place, and multyplye that 1 by 2, and
there amounteth 2, whyche I sette in the
fyrst place, and thanne doothe the summes
stand thus.

1365
236
13330
6826
1115
3980
1100
2620
322140

And so haue I ended the whole
multiplicatiō. But now (as you
taught me) to know wyat thys
hole summe is, I muste adde all
thole parcels togyther, and then
vnder the lyne wyll appeare the
grosse or totall summe, that is
322140. M. That is well done
S. Then (me thinketh) I would
calle it well doone, whan I knowe whether
I had well doone or no. Maist. It may be
tried by 9, as addition was, but the surest
profe is by Dyuision, and therfore I wyll re
serue that, tyll you haue learned the art of di
uision. And before we passe from Multiply-
cation, I wyll yet shewe you an other maner
of multiplication, whych are counted of som
men, both more ready and more certayne, of
whych the one differeth nothyng frome thys
that I haue taught you, saue yt it dooth vn-
derstande all wayes the artycles, and ioyne
theym to the nexte summe, and therefore
I wylle declare it onely by an exaumple:
If I woulde multyplye 1542, by 365.
I muste sette theym as I sayde before,

and than doo I multiplye 2 by 5, and it maketh 10, of whiche I write the article vnder the fyrst place, and kepe the vygite 1 in my mynde.

Than say I foorthe: 5 tymes 4 doo make 20, and the one in my mynde, are 21, thereof I wryte the 1 vnder the second place, and kepe the 2 in my mynd

Than come I to the thyrd fygure, 5, sayinge: 5 tymes 5, make 25, and the 2 in my mynde, make 27, whereof I write the 7 vnder the third place and kepe the article 2 in my mynd.

Than comyng to the laste figure I saye: 5 tymes 1, make 5, and 2 in my mynde make 7, that dooe I write vnder the fowerth place.

And than haue I ended my fyrste multiplier. Than doo I lykewyse vnto the second multiplier, sayinge, 6 tymes 2 make 12, thereof I wryte the vygite 2 vnder the seconde place, and kepe the article 1 in my mynde.

Than saye I foorth: 6 times 4 maketh 24, & the 1 in my munde make 25, so I set that 5 vnder the thyrde place, and kepe the 2 in my mynde.

Than multiplye I foorthe, saying: 6 tymes 5 maketh 30, and 2 in mynde make 32, whereof I wryte the 2 vnder the fourthe place

1542	1542	1542
365	365	365
10	710	7710
		2

place, & kepe the 3 in my mynde. Then doo I multiply the last figure 1 by 6, and it maketh 6, to that I adde the 3 in my mynde, & it maketh 9, which I write in the v. place

1 5 4 2
3 6 5
7 7 1 0
9 2 5 2

And so haue I ended. ij. figures of the multiplyer. Than with the thirde and last multiplier do I lyke waies, and saye fyrst, 3 tymes 2 make 6, which I write in the 3 place vnder the multiplier. Then by that 3 doo I multiplye lyke waies the second figure 4, & it maketh 12, wherof I write the digite 2 vnder the fourth place, & the article 1 I kepe in mynde.

1 5 4 2
3 6 5
7 7 1 0
9 2 5 2
2 6

Than come I to the third figure 5, sayinge: 3 tymes 5, maketh 15, and the 1 in my mynde make 16, thereof I write the 6 vnder the fyfte place, and kepe the article 1 in my mynd.

1 5 4 2
3 6 5
7 7 1 0
9 2 5 2
6 2 6

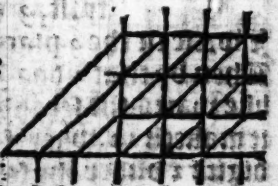
Than come I to the laste figure whiche is 1, and multiplye it by 3, and it maketh 3, thereto I adde the 1 in my mynde, and it maketh 4, whi-

1 5 4 2
3 6 5
7 7 1 0
9 2 5 2
4 6 2 6

che I writ in the vi. place. And thā haue I ended the multiplication, and the fygures stande in order thus.

1542
365
7710
9252
4626
562830

which parcels if I adde into one summe, it will be 562830, whiche is the grosse or treall summe of all that multiplication. Sco. Wel, this maner of multiplicatyon I perceaue: but what other sortes haue you? Mayster. Ther is one waie that is wrought by a checker table made thus.



And looke howe many places your sum hathe that you wold multiply so many squares muste you make in your table: from the ryght syde to the lefte: and so many from the hygher part to the lower, as there be places in your multiplier. Then set down your greatest summe firste on the top of the table, euery figure in due order in a square alone, I meane in those squares, that be opē and uncrossed. And likewaies in those lyke squares at the right hande, set downe your multiplicatour or multiplier, the last figure in the highest place, and so downeward, that the fyrst figure may be in the lowest place. Scholar. Syr, if it please you, me thynketh than I understande you best, whan you doo not stand long in tellyng the rule before examples, but propose some example, and then in declaryng it, bryng in the rules withall. M. In dede that way is easiest for a yōg lerner, therfore will I euen so do. Take this ex

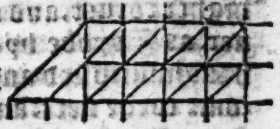
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ample nowe: I wold multiply 2036 by 23. Fyrst I consyder that my greatest numbre hath 4. figures or places, and therefore I make so many roumes betwene lynes, thus.

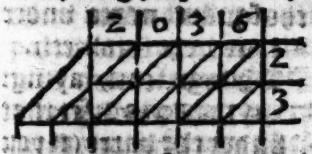
Then I se that of my multipliers there ar ij, wherfore I draw so many lines a crosse ỹ other, that ther may be 2 romes betwen thẽ, thus.



But you must not forget to let the endes of the lynes runne out (as it appereth in this patron) for in those open squares must your ij. first numbres, and al the totall summe be set. Then draw a crosse barre thorough euery close square, so that it may reache downe to the lowest ouerthwart lyne, as in this fourme.



And then is your checker forme prepared: thã set downe your fyrst or greatest summe on the top, & your multiplier on the right syde in the open squares thus.



Than begin to multiply the firste figure of the higher sum by the hyghest of the multiplier, saying: 2 times 6 make 12, that 12 must you write in the square ỹ is against the 2 and the 6, but of suche maner that the digette bee sette in the nether corner of the square, and the article in the higher corner,

as you may see in this example.



And so of euerye other multiplicatiō, whatsoeuer amounteth you must wryte in the common square which is agaīst bothe those figures, by whiche you doo multiplye. and if that summe do make but one digite, than must it be sette in the lowter corner of the square: but if it make an article, than wryte the article in the hyghest corner, and let the cypher go (of ye wyll) euermore, for here it serueth for nothyng, seyng the lynes doo distincte the places, but if the summe amountyng of such multiplicatiō do make a mixt numbre, then wryte the article in the highest corner, and the diget in the lowter corner, as I dydde by that 12: then whan you haue multiplied and ended the fyrste figure, come to the nexte. and multiply it in like maner, as in sayinge: 2 tymes 3 is 6, that 6, because it is but a digette, you shall sette in the nether corner of the square, nexte vnder



3, as here followeth. Then go forth saying: 2 tymes 0 is 0. set that vnder the barre (if you list) in the third square. Than forth. I say 2 tymes 2 make 4, that set in the last square vnder the bar, so haue you



ended

ended ỹ firſt multiplier: come therfore to the ſecond & ſay: 3 times 6 make 18. of whiche ſum the article 1 muſt be ſet aboue the barre, in that ſquare that is next to the 2 (as you ſe here) & the 8 vnder the barre. Then ſaie: 3 tymes 2 maketh 6, ſette it in the nexte ſquare beneth the barre, thā 3 times 0 is 0, writ it in the next ſquare or let it go, for all is one. & I perceiue it wel, for here the lines diſtinct the places, wherfore cyphers doo onely ſerue & therfore here they needе not to be. M. Than ſaie farther, 3 tymes 3 make 6, write that in the laſt ſquare, then wyll the hole figure ſtād thus as here followeth.





& Now could I me ſemeth doo lyke agayn: But howe ſhal I do now to gather the ſumme? M. Marke fyrſte the order of the places in this figure, and ſo ſhall you perceaue the reaſon of gatherynge them into a ſumme. The ſloye barres doo part the places, ſo ỹ the fyrſt place is the loweſt corner (in all ſuche figures) of the nethermoſte ſquare, next the right hande, and all the halfe ſquares betweene that barre and the nexte, ſtandeth for the ſeconde place, and ſo the roume betweene that & the nexte barre is the thyrde place, and ſo forthe. Nowe if you perceaue this, then muſte you adde all

the figures of one place together, as if you had an addition of diuers summes. S. If I vnderstande you rygbte, then must I take bers in this example 8 to be in the fyrst place 9, 1, 8 2 in the seconde. 0. 6. 1, in the third, 6, 0, in the fourthe, 4 in the fyfte, and the fyxte place hath no figure. M. you saye well. and the reason is, because the multiplication serueth to that square made but a digette, S. Then it is all one, as if they stoode thus. M. Euen soit is, and now adde this summe, and there wyll appete the totall of the multiplication to be 46828. And if you wyll se the agrement of this maner of multiplication, and the other that you learned before, then multiply those two summes (that is 2036 by 23) after the first maner without squares. S. You mean to set the thus in order And then multiplie 3 into 6, make 18, 3 tymes 3 make 9, 3 tymes 0 is 0, then thre tymes 2 make 6; whiche must be sette thus.

	2036
	23
	18
609	
12	
406	
46828	

Then do I lyke wais with the second multiplier, saying: 2 tymes 6 make 12, 2 tymes 3 make 6, 2 tymes 0 is 0, and 2 tymes 2 make 4, whiche what I addde to the other, then wyll the hole multiplication stand thus.

	2036
	23
	18
609	
12	
406	
46828	

Maist

Maist. So that you may see in euery place the same figures here, as they were in the multiplication by squares, though they differ in the height and lownes of places, but being added togither, they make one summe. And thus nowe ye haue learned iiii. formes of multiplicatiō, which you like best, that may you vse. Yet are there other formes, but litle thei nothing differ frō these three in effect, but only in setting of the numbres, I wil ouerpasse the cal a more meeter place & tyme. And now wil I instruct you in Diuisiō, so you thinke your self sufficiētly to perceiue what I haue taught you. S. Yes sir, I thāk you, but I do not perceiue howe to examine my worke, to try whether I haue wel don or no. M. That is cōmonly vsed by the proff of 9, as you learned before in Addition & Subtraction, saue that it hath this waies diuers from thence, first you must make a cros after this maner. Then must you examine your summe that shoulde be multiplied, and lose what remaineth after casting away of 9, that set you at the one side of the crosse then examine the multiplier, and what lo-uer remaineth in it, after casting awaye 9, as often as you can, write that at the other syde of the crosse: then must you multiplye those ii. numbres togyther, and lose what amounteth thereof, if it bee vnder 9, write it at the vpper parte of the crosse: but yf it bee

The proff of multiplication.

be aboue 9, then take thence 9 as often as you can, and write the reſt at the head of the croſſe, as in the laſte example of multiplication, the numbre to bee multiplied is 2036, wherin is ones 9, and 2 remayneth, whiche I write at one ſyde of a croſſe, thus. Than doo I examyne the multiplier whyche is 23, wherein there is no 9 but 5 in all, that 5 therfore I ſette at the other ſyde of the croſſe, thus.



Then do I multiplye 5 by 2, and it maketh 10, frome whiche I withdrawe 9, and there reſteth 1, that 1 do I ſet at the head of the croſſe, than do I examine the groſſe ſumme amountyng of the multiplication, whiche is 46829, wherin I fynd 9 thre tymes, and 2 remaynyng, that 2 I ſet at the tote of the croſſe, and then I ſe it to agree with the other 2 at the top of the croſſe, and ſo know I that I haue done well: for if they two did differ, than were my worke vain, and the multiplication falſe: This is the cõmon prote, but the moſt certayn prote is by Diuiſion, of whiche I wyll anone inſtructe you. S. Syr what is the chiefe vſe of Multiplication? M. The vſe of it is greater then you can yet vnderſtand, howbeit theſe plain commodities it harbe, that if you wolde reſolue any great and hole valure into manny ſmall & leſſe portions: as if you wold change pounde into ſhyllynges, peunies, or any o-

ther greater or smaller parcels, by multiplication ye shall do it spedyly and easyly. Also yf you shoulde neede to adde one summe to it selfe or to anye other occurrynes, you shall do it by multiplication much more spedely, readily, easyly and surely, than by often and sundry Additions: Take you these commodities grossely shewed for an answer at this tyme, and here after I wyll more abondantly make you to perceaue the vse of it.

DIVISION.

Scholar.

WEl syr, then in Division I pray you to instructe me. But me thinketh by the name of it, that it shuld be all one with Multiplication: for I call that Division, when any thyng is parted into dyuers and many partes. Mayst. You take it as it is taken commonly, how be it, if you marke well, you shall perceaue that it is quite contrarye to Multiplication, and dooth not parte one thynge or fewe thinges into many, but contrary wayes it bryngeth many parcels into fewe, but yet so, that these fewe taken togy-

there

ther, are equall in valure to the other many, for by dyuysion pence are tourned into shyllynges, and shyllynges into poundes: as for example of 120 s, it maketh 6 li. So are 120 tourned into 6, which is a smaller numbre, but then if you conspyre the denominatours, you shall se that thei ar suche, that one of the later is equall to 20 of the fyrst, and so in valewe the summes ar one, thoughe in numbre they doo varre dyuers, and the later sum is the lesser, & so is it always in Dyuysion, how be it yet, in the worknyng the sum is parted by an other, & therof doothit take the name. Scholar. I thynke I shall better understand the reason of the name, when I know the vse of the worke, therefore now woulde I gladdely learne that. Master. Diuision is a partition of a greater summe by a lesser, therefore (as you may perceaue) vnto Diuision ar required two numbres, the fyrste, whyche shoulde bee diuided, and that must be the greater, and the seconde, by whyche the other muste be diuided, and that is the lesser, and is called the diuisor. The first must be fyrst written, & the seconde so set vnder yt, that the last figure of the lower numbre be right vnder the last of the vpper, contrary ways to the worke of the other kindes of Arithmetyke, for in them the two fyrst figures wer set euer meet one vnder the other, but in Dyuision the last figures must be set

meete

mete, excepte it chance so that the last figure of the Diuisor be greater then the last of the hygher numbre, for then you shall lette the last of the Diuisour, vnder the laste saue one of the hygher numbre, as for example: if you shulde diuide 365 (whych are the summe of the daies of a yeare) by 28 (whyche are the dayes of a common moneth) then shuld you set them thus.

365
28

But if you would deuide those 365 dayes by 52 (which is the numbre of weekes in one yere) then should you set them thus.

365
52

Lykewyse yf I wold deuide the same 365 by 4, which is the summe of the quarters of a yeare, then must I set the thus

365
4

S. Syr thys do I vnderstande, but how now shuld I do to deuide the one by ỹ other? M. You must begi with the last figure next the left hād, & se how many tymes the last figure of the diuisor may be take out of the last figure of the ouer numbre, & ỹ shal you note within a crooked lyne toward your right hand, as for example: I would deuide 365 by 28, then set I those 2 summes, thus:

365
28

And I loke how many times I may fynde 2 (whych is the laste figure of the diuisour) in 3 (which is the laste of the numbre to be deuided) and considerynge that I can take 2 out of 3 but ones, I make a croked lyne at the ryght hand of the numbres, and within it I set 1, and that is

called

quotient nombre.

called the Quotient numbre: then bycause that when 2 is taken out of 3, there remayneth 1, I must wryte that one ouer 3, and deface and cancell the 3, and the 2, then wyll the figures stand thus. 1

Then muste I go to the next figure of the deuisor, & take it likewise so many times oute of the figures ẏ be ouer it, & loke what booth remayn, that I must wryte ouer thẽ, & cancel thẽ, as in this exaple: I take ones 8 oute of 16, & there remayneth 8, which I must set ouer the 6, and cancell or crosse oute the 16 and the 8 of the diuisour: And then will the figures stand thus. And so haue I ones wrought. So I perceaue ẏ you take the nether figure not only. 1 ✠ 8 ✠ 8 ✠ 8

take your quotient lesse by one, & then proue agayne, and so styll tyll you fynde a meete quotiente. When you haue thus wroughte ones, than muste you begyn agayn, & wryte your diuisour a new, nerer toward the right hande by one place, as in this example: you shall sette 2 vnder 8, and 8 vnder 5, thus.

```
x 8
3 6 5   (1
x 8 8
  2
```

Thā (as before) seke howe manye tymes you maye take the laste diuisour out of the numbre ouer hym. 8. That may I do here 4 times. M. Truth it is, that you may fynde 2 foure tymes in 8: but then marke whether you can fynde the figure folowyng so many times in the other that is ouer hym. Can you fynde 8 foure tymes in 5? S. No, nother yet ones. Maister. Therfore take 2 out of 8 ones ye le. Scho. That is 3 tymes. Mayst. Well, then 3 times 2 make 6: yf I take 6 out of 9, there remaineth 2, which 2 with the 5 folowyng make 25, in whiche summe I may fynde 8 thre tymes also, and therfore I take 3 as a true quotient, and wryte it within the crooked lyne of the quotient before the 1: thus.

```
x 8
3 6 5   (13
x 8 8
  2
```

Than say I: 3 tymes 2 make 6, than 6 out of 8 resteth 2, therfore I cancell the 2, and the 8, and wryte ouer it the 2, that dothe remayne, thus.

```
x 8 8
3 6 5   (13
x 8
```

Thenne dooe I take 8 as many tymes out of 25, sayenge: 3 tymes 8 make 24, and yf I take 24 oute of 25, there remayneth 1, so then I cancell 25 and 8, & ouer the 5 I set 1, thus

x		And now haue I done
x 8 1		with diuydynge, for I
x 6 4	(13	can fynde my diuisour
x 8 8		28 no more in the ouer
x		summe. S. No, excepte

you woulde parte the 1 that remaineth into 28 partes. Mayster. That is well sayd, and so must we dooe in suche cases, when there remaineth any thynge: but I wyll let that passe now, & wyl make you perfect in hole diuision, and will hereafter teache you peculiarly of broken noumbre called Fractions. And now if you doo perceiue the order of diuision, thenne doo you diuide this summe 136280 by 452. S. Fyrst I sette downe the numbre that shoulde be diuided. than do I set the diuisor vnder it, so that the last figure of it bee ryghte vnder the laste figure of the ouer nūbre. Thā wyll it be thus. 136280

M. Can you take the last of 452

your diuisor (whiche is 4) out of 1, whyche is the last of the ouer numbre? S. I had forgoren, bicause the last of the diuisor can not be taken out of the last of the ouer numbre, in as muche as it is the greater, therefore must I sette the diuisour one place more forwarde

ward to ward the ryght hand, 1 3 6 2 8 0
thus. And than must I looke 4 5 2
how often I may fynde the last figure of the
diuisor (that is 4) in 13, which thing I may
doo 3 tymes, therfore doo I say, 3 tymes 4
is 12, whiche I take out of 13, and there re-
maineth 1: than do I make at the right hād
of my summes a croked lyne, and write be-
fore it my quotient 3, and I cancell 13 and
4, and ouer the 3 I sette the 1 that remay-
neth, and then the figures stande thus.

1 — Then doo I multiply the
x x 6 2 8 0 (3 same quotient into euerye
4 5 2 figure of the diuisour, and
withdrawe the summe that amounteth out
of the numbres ouer them: as ryght I say:
3 tymes 5 make 15, whiche I take from 16,
and there resteth 1, I cancell therefore 16
and 5, and write ouer the 6 that 1 that re-
mayneth, thus. x 1

Than doo I say lyke x x 6 2 8 0 (3
wais: 3 tymes 2 make 4 5 2
6, which I take out of 12, and there resteth
6, therfore I cancell the 12 and the 2, and
ouer the 2 I write 6, that remayneth, thus.

x x 6 Then shuld I set forward
x x 6 x 8 0 (3 the diuisor into the nexte
4 5 2 place towarde the right
hād thus. M. But you wat x x 6
se, that ouer the 4 is no x x 6 x 8 0 (3
figure, therfore must you 4 5 2
set the diuisor yet forwarder 4 5 2

by an other place. And marke whā so euer it chāceth so, that you shuld set forward the diuisor, & that it can not stande there, bycause there is no numbre ouer the laste place: or if ther be any, it is lesser than the last figure of the diuisor, thā must you remoue the diuisor yet ones again: & bicause that his first place of remouyng fayled hym, therfore shall you write in the quotient a cyphre 0, and if you shuld by chaunce nede to doo so often tymes, for euery time write a cyphre in the quotiēt. The reason of this will I shew you hereafter. S. Then muste I set my summes thus.

```
  x x 6
x 3 6 2 8 0  (30
  4 5 2 5 2
    4
```

And bicause I remoued the diuisour, so I ouerskypped one place, I must write a cyphre in the quotiēt: and than must I seke a newe quotient, as in this example, I must say: How many tymes 4 is there in 6? and sithe it can be but ones, therfore do I write 1 in the quotient, and then say I, 1 tymes 4 taken out of 6, remayneth 2, I cancelle the 6 and the 4, & write 2 ouer them. Then say I again: ones 5 out of 25, remaineth 23, I let the 2 stand as it dyd, and ouer the 8 I set 3, cancellyng the 8 & the 5 vnder it, thus.

```
    2
  x x 6 3
1 3 6 2 8 0  (301
  4 5 2 5 2
    4
```

M. You myght as wel haue said, ones 5 out of 8, & so remayneth 3, but nowe go forth.

Schol.

8. Than ones 2 out of 0, can not bee, what shall I now doo? May. Borowe of the nexte numbre that is behynde (for there is 230) & doo as you lerned in Subtraction in a lyke case. S. Than must I borow one of the 3 comyng behynde next, and make that 0 to bee 10, and then take I 2 oute of 10, and there resteth 8, and bycause I borowed 1 of the 3, I must cancell the 3, and wryte 2 ouer it, than dooth the fygure stand thus.

2 2
1 1 6 3 8
1 3 6 2 8 0 (301
1 3 2 2 8
1

May. Nowe haue you doone, and yet remayneth 228, and youre quotient dooth shewe you, that yf you dyuide 136280 by 452, you shall fynde your diuisor in your greater numbre 301 that is, CCC. tymes and ones, and 228 remaynyng. And in the other example where I deuyded 365 by 28, the quotient was 13, and 1 remayned, wherby I know that in a yere (whiche contayneth 365 dayes) there are 13 monethes, reckenyng 28 dayes (or 4 weekes) iust to a moneth, and 1 day more. S. Why than doo we call a yeare but 12 monethes? M. Of that at a more conuenient tyme wyll I fully instruct you, but nowe it is not conuenient to intangle your mind with other things, than to directly pertayne to your matter. therfore yf you can remembre what you haue herd, you haue lerned a short maner of diuision, which

I wouldde haue you often to practyse, so that you may be perfect in it, and hereafter I wyl shewe you certayn other propre pointes touchyng it. Sch. Than I pray you yet tell me, howe shall I examine and trye my woorke whether I haue don wel or no? that thoughe no man be by me to tell me, yet I may perceaue it my selfe. Mai. Some men (yea and cõmonly) do trie that by the rule of 9, as in all the other kyndes, saue that their order is this. Fyrste they caste awaye 9 as often as they can out of the deuisor, & that that remaineth, they sette at one syde of a crosse, as in our fyrst example the diuisor was 28, from whyche you may take 9 ones, and 1 remayneth, which they set by a crosse thus.

Than do they lykewyse examin the quotient (which in our exãple is 13) & frõ thẽs they cast away 9 as often as they can, & the remainer they set at the other syde of the cresse, & than multiplye they together those 2 remainers, & to it that amounteth, they adde the remainer of the diuisiõ, if there were any, from that hole summe they withdrawe 9 as often as they can, & the rest they set at the head of the crosse, as in our example the quotient is 13, whiche maketh only 4, and therefore you must set 4 at the other syde of the crosse, thus

Then multiply 4 by 1, & it yeldeth but 4, therto ad the remayner of

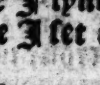
the proofe of diuision.

the

the diuision (which was 1) & it will be 5. why the sum doth not amounte to 9, and therfore must be set holly at the head of the crosse, as you see here. And this numbre on the head of the crosse is the fyrst prefe, to which if you find an other like in the numbre that was diuided, thā haue you don wel. Therfore now shal you likewise examin the hole summe that was diuided, and take out 9 as often as you can, & that that remaineth set at the foote of the crosse, & if it be equall to ỳ in the head of the crosse, than haue you well done, els not: as in our example the hole sum was 3,6,5, which maketh 14, frō ỳ take 9, & ther resteth 5, which set at the foot of the crosse, thus. And you shal see ỳ they agree, therefore haue you wel done. S. Now will I likewise examine our second example, where the diuisor was 4,5,2, which maketh 11, frō thēs I take 9, & the 2 that remaineth, I set at the right side of a crosse, thus. Then examine I ỳ quotient whych was 3,0,1 where I find but only 4, that do I set at the other side of ỳ crosse, thus. Than doo I multiply 4 by 2, and it maketh 8, to that do I adde the remainer of the dyuision (which was 2,2,8, & maketh 12) and the two make 20, wherein I fynde twyse 9 and 2 remaynyng, that 2 must I set at the head of the crosse, thus.



Than do I examine the whole nũ-
bre to bee dyuyded, whyche was
136280, where I fynde twyse 9
and 2 remaynyng, whyche I set at the toote
of the crosse, thus.



And bycause that it dooth agree
with the figure at the head of the
crosse, I knowe that the diuision
was well wrought. S. This is the common
profe, how be it, the more certayne working
is by the contrary kynd, as to proue Diuis-
on by Multiplication, thus, Multiplye the
quotient by the Diuisour, and yf the summe
that amounteth be equall to the summe that
shoulde be diuided, then haue you well dy-
uided, elsi not. Howe be it, this muste you
marke, that if there remained any thing af-
ter the diuision, that muste yee adde to the
sum that amounteth of the multiplicatiõ: as
in our fyrst example the quotient was 13, &
the diuisor was 28. Now multipli the one bi
the other, & the sum wyll be 364, to that yf
you adde the 1 that remayneth after the diui
sion, than wyll it be 365, which was the sum
that shuld be diuided, and therfore I knowe
that I haue well done. S. Now wil I proue
the same in the second example, whose diui-
sor was 452, and the quotient 301, theke do
I multiply togyther, and there amounteth
136052, to which if I adde the 228 yͤ remai-
ned, thã wil it be 136280, which was yͤ hole

summe

summe to be diuided. & therfore I perceaue that I haue well doon. M. Thys is the surest waye to examine Diuision by Multiplication, and contrary wayes, the sureste proofe of Multiplication is Diuision. And therefore nowe wyll I shew howe you maye proue Multiplication by Diuision. When you haue ended Multiplication, and wolde know whether you haue wel done or not, set the grosse summe that amounteth of the multiplycation outwarde, and deuide it by the multiplier, and if the quotiente be the same numbre that should be multiplied, then haue you wel wrought, els not: as in that example, where wee multiplyed 264 by 29, the grosse summe was 7656. Now if you wyll know whether that multiplycation be true, you shall deuyde that 7656, by the multiplier. 29. And you shall perceaue that the quotient wyll be 264, and that is a token that you haue wel wrought. S. By your patience I wyll proue that, and fyrst I set downe the grosse summe, and the multiplier, not after the rule of Multiplication, but after the rule of Diuision, for now that numbre is becom the diuisor, that was before the multiplier. I shall setre theym therfore thus. 7656 29

And then shal I seke how many times 2 in 7, that maye be 3 tymes and 1 remaineth, but then may not 9 bee founde so often in 16, therefore muste I take a lesse

quotyent, that is to say 2, then say I twyse 2 maketh 4, whych I take out of 7 & there remaineth 3, then doo I cancell 7 and 2, & ouer 7 I wryte 3, and in the quotient I set 2, so the figures stand thus. 3

Then saye I forth, 2 tymes 9 make 18, whyche I take out of 36, and there resteth 18, then cancel I 3, and ouer him set 1, and likewyse I cancell 6 and 9, and ouer them I set 8, so stande the figures. 1

Then do I set forward the diuisor by one place, and seek a new quotiēt, that is to saye, howe many tymes 2 are in 18, which I fynd to be nyne tymes, but then can I not fynde 9 so many times in 5, therfore I take a lesser quotient, as to say 8, but yet is that to great, for if I take 8 tymes 2 out of 18, there remayneth but 2, & I can not fynde 8 tymes 9 in 25, therefore yet I take a lesser quotient, that is 7, which is also to greate, for if I take 7 tymes 2 out of 18, there resteth 4, but now I can not take 7 tymes 9 out of 45, therfore yet I seke a lesser quotient, as to say 6, then say I, 6 tymes 2 make 12, that I take out of 18, and there remayneth 6, so I cancel that 18 and the 2, and wryte 6 ouer 8, then saye I forthe: 6 tymes 9 maketh 54, that take I out of 65 and there remaineth 11, and the figures stande thus,

Th

Then must I set forth ye diui
for again, & then take a new
quotient, which wyll be 4,
for though be I may fynd 2,
in 11 fiue times and one re
mayn, yet I can not find 9
so often in 16, therfore I set the figures thus.

1	And the 4 in the quoty-
26	ent I multyplye into the
781	fygures of the dyuisour,
7656(264	sayinge, 4 tymes 2 ma
2999	keth 8, whych I take out
2	of 11, and there reseth 3.

therfore I cancell the 11 & the 2, & set 3 ouer the fyrst place of 11, thus.
And then do I say forth, 4
tymes 9 maketh 36, whi
che I take from 36, and
there remaineth nothyng
so that the quotiēt of this
diuision, where 7656 is diuided by 29, is
264, which doth declare, yt 264 be multy-
plied by 29, ye sume wyl be 7656. And thus
I perceiue now how both Multiplicatiō is
proued by Diuision, & Diuision also by mul
tiplication M. Howe haue I ended the v.
most common kindes of Arithmetike, for as
touchyng Mediation, Duplation, Tripla-
tion & suche other, they are no seuerall kyn-
des of Arithmetike, but are contayned vn-
der the other, for Mediatiō is contained vn-
der

der Diuision, and is nothyng els but denydyng by 2: and so are Duplacion and Tryplacion contained vnder multiplication. for Duplatiō is nothing els but multiplying by 2, & triplatiō is multiplieng by 3. of which I wil only propose exāples, for ye rules you haue herd alredy, if you wold mediate or diuide into 2 this sum, 45310 10, you shal set 2 for ye diuisor, & worke as you lerned before, as thus

Then I fynde 2 in 4 twoo fymes, therefore my quotient muste be 2, so I cancell 4 and 2, & remoue the deuisor forwarde, thus:

4 5 3 1 0 1 0 (2
2

Than agayne I fynde 2 in 5 twis and 1 remaynynge, so I wryte 2 agayne for my seconde numbre of the quotient, and cancell 5 and 2, and ouer 5 I set 1 thus.

4 5 3 1 0 1 0 (22
2 2
1

Then remoue I the diuisor forward, and seke a newe quotient, whiche is 6, than say I, 6 tymes 2 make 12, take that oute of 13, and there resteth 1, so I cancell 2 and 13: & ouer 3 I set 1, thus.

4 5 3 1 0 1 0 (226
2 2 2
1 1

Then remoue I the diuisor forward, and seke a newe quotient, whiche is 5, than take I twyse 5 out of 11, and ther resteth 1, so I cancell the 2 and the last figure of 1, and let the fyrst stand thus,

Then

x x
4 5 3 1 0 1 0 (2 2 6 5
x x x x x

Then remoue I the diuisor forwarde, & seke a newe quotient whiche is 5, then take I 2 fiue tymes out of 10, and there resteth nothyng, then remoue I agayne the diuisour forward, thus.

But bicause I can not fynd the diuisor in the nũbre ouer it, I must set a cipher in the quotient, and remoue the diuisor to the next place thus.

x x
4 5 3 1 0 1 0 (2 2 6 5 5
x x x x x 2

Then seke I a newe quotient whiche I fynde to be 5, for so many tymes may I haue 2 in 10. Then haue I fully ended this Mediation or Diuision by 2, and the quotient is this, 2265505, which is the halfe of 4531010, as you may trye by Duplation: for double that quotient, or multiply it by 2, and the same numbre wil amount. But I wil no longer tary about these, seyng they are but membres of the other kyndes. But here now wil I teache you certain easy formes both of Multiplication and of Diuision, and fyrst of Multiplication. If you wolde therfore multiply any summe by 10, you shal nede to doo no more but adde a cypher before his fyrst place: as for example: 36 multiplied by 10, make 360. Like wayes if you will multiply any summe by 100, put

Easy formes.

two

two cyphers at his begynnyng. So yt you wold multiply any summe by 1000, adde iii. cyphers to the begynnyng of it. S. This doo I wel perceine, and also the reason of it. M. I will omitte all reasons, tyl our next metyng, when I shall tell you the reason of all other partes of Arithmetik also: and as to our matter, now I sake (as I haue told you) that you doo both remembre it, and also often practise it. But if you wold multiply any numbre by 5, marke firste whether the numbre be euen or odde, & if it be euen, take the halfe of it, and write a cyphre at the begynnyng of it, as for example. I wolde multiply 2564 by 5, I take the halfe of it, which is 1282 (as you may knowe by Mediation) and before it I sette a cypher thus, 12820, & this is 5 tymes 2564. And thus maye you doo with any other euen summe that you wold multiply by 5. But if the summe bee odde, as for example: 2563, then must you take the lesser halfe of it, or (yt ye wyl) take awaie 1 from the fyrst figure (as here frome 3) and then take the halfe of the rest, and set before it 5, as of 2563, the lesser halfe is 1281 for here I take but 1 for the half of 3: and if I put 5 before that lesser halfe, than haue I multiplied it 5 tymes, as thus, 12815. S. What meane you by the lesser halfe? M. There is no iuste halfe of any odde numbre, therefore if we diuide an odde noumbre into

two

twopartes as nygh equal as can be, yet will the one halfe excede the other, halfe by one, as for example: The two moste neresi halfes of 9 are 5 and 4, and lykewais of 15, are 7 and 8, where you see that the one parte styll is greater then the other by one. Nowe it is ealy to know which is the greater half, and which is the lesser halfe. S. Then I perceaue you, and can doo lyke waies (I doubt not) with any summe: for if it bee not very easy to part into halfe, then wyll I do it by Mediation easyly inoughe. M. That is a sure way. And nowe haue you learned how to multiplye easily by 5, 10, 100, 1000, and of lyke maner may you doo with any other of that sorte. But now yf you wil multiplye by 20, 30, 40, and so forthe, or by 200, 300, and suche lyke, where there is one cypher in the firste place, or made orderly in the fyrste places, you shall take away those cypheres, & multiply the sum onely by the other figure or figures (if they be many) and then at the begynnyng of the summe ſ amounteth, shal you set so many cypheres as you toke away. Example of 2873, which I wold multiplye by 300. First I cast away the two cypheres from the multiplier, & I multiply the sum by onely 3, that is lefte, and it amounteth to 8619, before whyche I putte the two cypheres, that I tooke awaie before, and then is it 861900. And that is the summe that a-

moun-

mounteth, when 2873 is multiplied by 306, 8. And if there were two or more figures beside the cyphres, I muste onely take away the ciphers, & multiply by the other figures, as I lerned before, as if I wolde multiplye 93648 by 25000, I shuld take awat the thre ciphers, & multiply the same by 25, and then at the begynnyng of that totall summe shuld I adde the thre cyphres again. M. Euen so, but and if it chaunce the numbre that shuld be multiplied, or bothe the summes, as well the numbre that shuld be multiplied, as the multiplier, to haue ciphers in their first places, euermore caste awaye the cyphers and worke by the reste, but remembre to restore as many cyphres to the amountyng summe as you bared before, as in this exãple. 30200 shall be multiplied by 206, I shall only take away the two ciphres from the greater nũbre, and then multiplye 302 by 206, and afterward adde the two ciphres again. Butif I wolde multiply the same 30200 by 2060, I shall not onely take away the two ciphers frome the numbre that shuld be multiplied, but also I maye take awaye the one cyphre from the multiplier, and then must I adde 3 cyphres to the summe that amounteth: but take hede that you take awaye no cypher ♀ commeth after any sygnifyeng figure, as in this laste example, you may not take awaye that in the .iij. place of the hyghest numbre, nother

nother that in the thyrde place of the multiplier: howe be it, yet this you may doo: If one cyphre or more come in the myddes of your fumes, you may multiply by the other fygures, and ouerskyppe theym, but so, that you gyue euery fygure his dew place, as thus: I wyl multiply 3026 by 2004 therfore I set theym thus.

3026
2004
24
1208

And thus I multiplye them: fyrst 4 tymes 6 make 24, I sette the 4 vnder the fyrst place, and kepe the 2 in my mynde, or wryte it downe for easy remembrance: than say I agayne: 4 tymes 2 maketh 8, and 4 tymes 0 maketh 0, then 4 tymes 3 maketh 12. but now when I come to the nexte cyphre, bicause that it multiplyeth nothyng, I let it go, and lykewayes the second cyphre, but then whan I doo come to the 2, and multyplie it into the 6 of the ouer numbre, you muste take heede (accordynge as I taught you in Multiplication) that the fyrste numbre amounting of the Multiplication, be set vnder the multiplier right, and the other orderly toward the left hand, accordyng as you may se in this exaumple.

3026
2004
24
1208
12
604

Where if you hadde expressed the cyphres after the common rate, than shulde the fygures stande thus.

But in effect all is one, saue that the fyrste waye, by ouer skyppynge of the cyphres, is the shorter and easyer way: for that in effect they be both one, the addition of the parcels wyll declare, whyche in bothe wyll appeare thus.

		3	0	26
		2	0	04
				24
	1	2	0	8
	0	0	0	0
0	0	0	0	
	1	2		
6	0	4		
6	0	6	4	1 0 4

and nowe will I make an ende of thys matter.

Scholar. Syr I thanke you, for I seg great ease in this wayes of multiplication, and if you can shew me lyke induction, you shall greatly further me.

Ma. Yes I wyll teach you some easy wates in diuisiō also, & fyrst thys: If I wold diuide any sum by 10, you shall onely with your pen make a square lyne betwene the fyrst figure of your summe and the seconde, and than haue you done, for the hole numbre that foloweth the lyne, stādeth for the quotiēt, & the figure that is before the line, is the remainer. as for example 3648 diuided by 10, wyl stāde thus: 364|8 Where 364 is the quotient, and betokeneth that so many tymes are 10 in 3648 and the 8 after the lyne, is the remainer, whi che can not be diuided into 10, but by breakyng it into fractions, wherwith I wyl not medle yet, And so lykewaies if you woulde diuide any summe by 100 with your penne, you shall cut away the 2 first figures: and if you

you wolde diuide by 1000, you muste cutte away the 3 fyrst figures: and so of any other diuisor, whose last figure is 1, and the other be cyphres, looke how many cyphers the diuisor hath, and so manye figures at the begynnyng shal you cut away with the squire lyne, and they stande alwayes for the remaynder, bycause they are lesse then the diuisour, and can not bee dyuided by it, and the other figures that bee behinde the lyne, stand for the quotient. But nowe if your diuisoure haue any other fygure in his last place, than 1, and in all his other places haue cyphres, looke howe manye cyphres there bee, cutte awaye so manye of the fyrste fygures of the numbre that shoulde bee diuyded, and diuide the reste that folowyth the lyne, by that fygure that is in the laste place, as if it were the whole diuysour: exaumple of 64284, whiche I wolde diuide by 300: here must I cut away the two fyrst figures (for so many cyphers my diuisour hathe) and must diuide the rest by 3, whyche is the fygure in the last place of the diuisour. Fyrst therefore I pare awaye the two fyrst figures, and the summe standeth thus, 642 |84. Than doo I dyuyde 642 by 3, and the quotient is 214, for in 6 I fynde twyse 3, and in 4 ones, an one remaynynge, whiche 1 wyth the 2 nexte before, dothe make 12, wherin I fynd 3 foure tymes; and this is a readye waye

to tourne shyllynges into poundes, for sythe 1 pounde dooeth contayn 20 shyllynges, I muste diuide the whole numbre of shillinges by 20, therefore easylye to doo it, I see that my diuisour hath 1 cyphre, and therfore I cut away one figure frome the begynnynge of the whole sum of shyllynges, and than I doo mediate or diuide by 2, the other figures or summe that foloweth. S. I wyl put an example: If you wolde deuide 64287 ℔, by 20, that is tosay: If I would tourne so many shyllynges into poundes, I muste cutte away the fyrst fygure, that is 7, and diuide the reste, that is 6428 by 2, so shall the quotiēt be 3214, wherby I know, that 64287 ℔ make 3214 li, & 7 ℔ remaynyng. M. Nowe proue by Multiplication whether you haue well doon or no. Sco. The quotient is 3214 whych I do multiply by the diuisour 2, and it dooth amounte to 6428. Matth. Hereby maye you perceaue not onely that you haue wel doon, but also how by dyuision you may tourne shyllynges easily into poundes. And contrary waies, by Multiplicatiō you may tourne poundes into shyllinges. But heere shall you se amongst dyuers men, dyuers formes of such diuision, but if you marke what I haue tolde you, you shall perceaue easylye all their wayes, for some men doo not cut awaye so many of the first figures of the sum that they wold diuide, as ther ar cyphers in the

the first places of their diuisor, but they sette all their cyphres orderly vnder the fyrst places of the numbre that they would diuide, and then with the other figure (or figures yf they be many) they diuide the reste of theyr summe. Exaumple. If they wouldde diuide 725931 by 3400, they sette theyr summes thus.

7 2 5 9 3 1
3 4 0 0

And then do they diuide ordrely tyl they come to the cyphres: for there they stay and ende their woorke, as in this exaumple: They seeke howe often 3 may be founde in 7, whiche is 2 tymes, and 1 remainyng, therefore they set 2 in the quotient and cancell 3 and 7, and ouer 7 they sette that 1 that remayned, thus.

7 2 5 9 3 1 (2
3 4 0 0
1

Then doo they go forthe, saying: 2 tymes 4 maketh 8, which they take oute of 12, and there remaineth 4, thus.

7 2 5 9 3 1 (2
3 4 0 0
4

Than renew they the diuisor forwarde, and seke howe often 3 maye be founde in 4, whyche is but ones, and 1 remayneth, than set they 1 in the quotiente, & cancell 3 and 4, & vnder them thet set that 1, thus.

7 2 5 9 3 1 (21
3 4 0 0
1

Than take they ones 4 out of 15, and there remeth 11: or elles more easyly: take ones 4 out of 5, and there remeth 1, so they cancelle the 4 and

3, and sette 1 ouer theym thus.

```
   3
  x 4 1
  7 x 4 9 3 1 (21
  3 4 4  00
    3
```

Then se they foorth the diuisor agayn, and seke how many tymes 3 are in 11, which they fynde 3 tymes and 2 remaynyng, so they set 3 in the quotient, and cancell 11 and 3, and ouer them setteth 2, thus.

```
    x 2
  x 4 x
  7 x 4 9 3 1 (213
  3 4 4 00
    3 3
```

Then doo they multiplie 4 by 3, whiche maketh 12, that withdrawe they oute of 29, and there resteth 17, of whiche the 7 muste be sette ouer the 9, and the 1 ouer 2, thus.

```
                 1
  And nowe are the 2 cy-   x x
  phers next ensuyng, so   x 4 x 7
  that the diuisor can no  7 x 4 9 31 (213
  more be sette forward,   3 4 4 4 00
  and therfore is the di-  3 3
  uision ended, and the remaynet is 1731.
```

Nowe the quotient, whiche is 213, doth declare, yf you diuide 7 4 5 9 3 1 by 3 4 0 0, you shall fynde it therin 213 tymes, and there remayneth 1731, so shall you fynde it, yf you worke as I taught you, by cuttyng away the 2 fyrst figures, bycause of the 2 cyphers, but this muste you marke (as you maye perceaue by this last exaple) that if there be left any other remaynet in the summe that was beyond the square lyne, that remaynet

muste

muste be sette to the later end of the fyrst remayner, whiche was cutte awaye with the squire lyne: as if you wold diuide 725931 by 3400, after the fourme that I taughte you, than would your summes appere thus.

```
   1                 So that 17, which re-
    * *              mayneth after the line
  * * * 7            must be sette to the 31
  7 2 5 9 3 1 ( 213  (that was cut awaye
  3 4 0 0            with the line) in higher
    * * *            places, as you se here.
    5 3
```

where that 17 with the 31 doo make 1731. And here wouldе I make an ende of Diuision, sauyng that there cometh to my mynd one late inuencion of easy Diuision, whiche I will brefely set forth to you, so that if you fynde ease in it, you maye vse it. Bycause that the hardest poynte in Diuision is the ready and easy fyndyng of the quotient nūbre: and agayne, if that be trewely knowen, all the rest is but lyght to bee doone, therefore this waies shall you quickely and truely fynde the quotient. Fyrste write the .ix. figures of numbre, I meane 1 2 3 4 5 6 7 8 9, not alōg as I haue set them nowe, but vp & downe, as in this fourme. And at the lefte syde of them drawe a longe lyne, as you se here: then cōsider the diuisor by whiche you entende to woorke, and sette it on the lefte syde of the longe lyne

1
2
3
4
5
6
7
8
9

tyght agaynst 1, and for a distinction draw a lyne vnder it, then multiply your diuisour orderly, by eche of those figures, begynnyng with 2, and so go downewarde tyll you haue ended all, and looke what dothe amount of the multiplication of eche figure into the diuisour, that doo you write agaynst the figure, whereby you dyd multiplie. Schol. By example I may perceaue it better. Maister. Take this example, 263845 to be diuided by 64, thā must I set the 9 figures, as I sayd before, and the diuisor muste I set agaynste the 1, with a lyne drawen vnder, thus.

64	1
	2
	3
	4
	5
	6
	7
	8
	9

Than muste I multiplye that diuisour by eche figure orderly: First by 2, and it maketh 128, whyche I must set agaynst 2 at the left hand. Than multiplye I 64 by 3, and it maketh 192, whyche is sette agaynste 3. Thenne 4 tymes 64 make 256, that sette I by 4. Then saye I: 5 tymes 64 make 320, that sette I agaynste 5. Than 6 tymes 64 make 384, that sette I agaynste 6. Then 7 tymes 64, make 448, whyche I sette agaynste 7. Further I saye, 8 tymes 64 make 512, whyche I sette by 8. And laste of all I saye: 9 tymes 64 make 576, whyche I sette agaynste 9. And thanne they wylle stande thus.

And

64	1
128	2
192	3
256	4
320	5
384	6
448	7
512	8
576	9

And so is the Table ended, by whiche you may easily fynde the the quotiente, as you shall see by exaumple nowe. Doo you sette downe the noumbres (as you learned before) accordynge to the order of Diuision. Scholer. That is thus. 263845 64

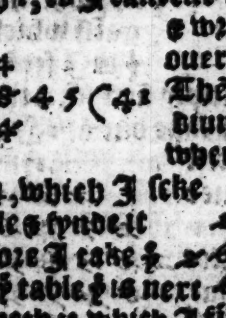
M. Nowe loke what numbre standeth ouer the diuisor, reckynyng therto al them that be behynde it towarde the left hande. S. Then are there ouer the diuisour 263. M. That is iuste. Nowe seke in the table on the left syde whether you cā find 263.8. It is not there. M. Then take that numbre that is nexte to it, benethe it: I meane a lesser numbre than 263, but of all the lesser numbres that the table hath, take you that, that goth nyghest to 263.8. That is 256. M. So is it: and mark this euermore, when you can not fynd iustely in the table that summe that is ouer your diuisor, then note that that is nexte benethe it, of any summe that is in the table, & loke at the right hande of the lyne, what fygure or diget that is agaynst that summe, & take that diget for your quotient, and than worke on, as you lerned before: for now haue I tolde you the hole vse of this table. Howbe it, yet that you may be sure to vnderstand it, I will se you ende this example of Diui-

tion by it. Nowe therfore begyn agayn. 8.
Firſt I ſet downe the ſummes after the com
mon manner, thus. 2 6 3 8 4 5
Then do I loke ouer the diui- 6 4
ſor, and fynde there 263. Now
to know howe many times 64 may be take
out of 263. I reſorte to the table aforeſayde,
and ſeeke for the numbre 263, but it is not
there, therfore (as you bad me) I take a leſ-
ſer numbre, the nexte to it that I can fynde
in the table, and that is 256, whiche num-
bre hath agaynſt it on the ryght hande this
digette 4, whiche I muſte take for the fyrſt
figure of my quotient. Than doo I (as I
learned before) multiply that quotient into
euery figure of the diuiſor orderly, withdraw
ing the ſumme therof amongſt ſtyng, out of the
oute ſumme, as here I ſay fyrſt. 4 tymes 6
make 24. ſo I take that out of 26 ſayeng: 4
out of 6 remayneth 2, whiche I write ouer
the 6, then 2 out of 2 remaineth nothynge,
then canceil I 2 and 6, and alſo 6 in the di-
uiſor, and the ſumme ſtande thus.

2 Then doo I likewyſe ſay
x 6 3 8 4 5 (4 fourth, 4 tymes 4 make
6 4 16, whych I take out of
23, & ther reſteth 7 to be
ſet ouer 3, and that 3 with the 2 behynde it,
& the 4 vnder it, muſt be x 7
cancelled, as you ſe here. x 6 3 8 4 5 (4
Thā haue I doon with 6 4

the

the fyrſt figure of the quotient. M. Now ſet forward your diuiſor, and ſeke a new quotient, as you ought this. S. Than thus ſtandeth the figures, ſo that ouer the diuiſor I ſe 78, which I ſeeke in the table, & can not fynde it, therfore I take the next leſſer, & that is 64, the diuiſour it ſelfe. M. So muſt you doo when there is none other. S. Than agaynſte it I find this diget 1, which I muſt ſet i the quotient before 4, thus. Then multiply 16 by 1, and it is but 6 fyll. M. You neede not goo about to multiply whã the quotient is 1, for 1 dothe nother multiplye nor diuide, but in ſuche caſe only ſubtract the diuiſor out of ỹ numbre that is ouer it. S. Than I take 4 out of 8, & ther reſteth, 4 & 6 out of 7, there remaineth 1, ſo I cancelle thoſe numbres, & write ỹ remayners ouer their place thus. Thẽ ſet I forward ỹ diuiſor agayn, thus. where I ſe ouer ỹ diuiſor 144, which I ſeke in the table & fynde it not, therfore I take ỹ numbẽ in ỹ table ỹ is next therto beneth it. which I fin-



to be 128. agaynste whiche in the right syde I fynde 2, whiche I take for my quotiente, and that doo I multiplye fyrste into 6, and therof commeth 12, which I take out of 14 and then remaineth 2, that 2 I set ouer 4, and cancel the other figures, 1, 4, & 6, thus.

```
      x 2
    x 7 4
x 6 3 8 4 5 (4 12
    6 4 4 4
      6 6
```

Then saie I forth, 2 times 4 are 8, which I take out of 14, & there remayneth 6, of which I write the 6 ouer 4, & the 1 ouer 2, & cancell 2, 4, & 4, thus.

```
      1
    x 2
    x 7 4 6
x 6 3 8 4 5 (4 12
    6 4 4 4
      6 6
```

Now again I set forward the diuisor thus. And seyng ouer it 165,

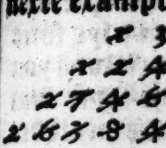
I seke ½ in the table, but fynd it not, therfore I take ½ nerrier ster, which is 128, against which I find 2 ½ wo I set into the quotet, & by it I multiply fyrst 6, & therof commeth 12, which I take out of 16, & there reseth 4, thā cancell

```
      1
    x 2
    x 7 4 6
x 6 3 8 4 5 (4 12
    6 4 4 4 4
      6 6 6
```

I 1, 6, & 6, & ouer 6 I set 4 thus. Then doo I multiply 4 by 2, & it maketh 8, which I take out

```
      x
    x 2 4
x 7 4 6
6 3 8 4 5 (4 12½
6 4 4 4 4
  6 6 6
```

out of 45, and there remaineth 37, as in the nexte example folowyng.



And nowe haue I doone. May. Well, now I see ỳ you can worke by this kind of diuision, as farre foorth as I taught you. S. Yea syr, I thanke you, and I fynde in it muche ease and certaynesse. M. Yet one thing I doubt whether you perceaue: What yf you dyd fynd in the table the numbre that standeth ouer the dyuisor, what woulde you nexte do? S. I thynke I shulde take the dygette agaynst it on the right hande for the quotient. M. So it is: and as often as you seeke in the table and fynd your numbre iust the dygette agaynst it, is your true and iust quotient. I call that a true quotient, if it be the right quotient ỳ you should take, though your diuisor multiplied by the same, do not clerely subtract the numbre ouer it, but there dooth somewhat remayne, as it chaunced in all the examples that you dyd worke by: but if it shoulde chaunce (as it dooth often) that your diuisour multiplied by youre quotiente, doo subtract cleane the numbre ouer it, then call I that quotient not only a trewe quotient, but also a iuste quotient, bycause it doth iustly consume the nũbre ouer the diuisour: ỳ that chaunceth euermore when the numbre

ouer

ouer the diuisor is iustly found in the table. S. Thys I shall remembre. Ma. But yet one easy poynt more I wyll tel you in thys sorte of Dyuision, therfore marke it wel: what you haue found in the table, other the same sum that is ouer the diuisor, other the nerte benethe (for lacke of the other) than look what diget standeth against it, take that for your quotient. and because it is some payn to multiply the diuisor by the quotiēt, you shall not nede to do it, but only take the numbre that you founde in the table, & subtract that from the ouer numbre: for yf you do multiply the diuisor by the quotient, that wyl be the numbre that shall amount. therfore is this waye more easyer. Sc. So is it, and also more certainer for such as I am, that might quickly erre in multiplying, especially beyng smally practised therin. M. The proue in som briefe example whether you can doo it. and so wyll we make an ende. S. I wolde diuide 38468 by 24, therfore fyrst I set the table thus. Then set I the two summes of diuision thus, 38468 And ouer the diuisor I find 38, which I seeke in the table & fynd it not, therfore take I the nert beneth it, which the table hathe, & that is 24 the diuisour it selfe, against which is set 1, which I take for the quotiēt, which I set in his place and

24	1
48	2
72	3
96	4
120	5
144	6
168	7
192	8
216	9

And nowe I need not to multiply the diuisor by it, but onely to withdraw the diuisor out of the 38 that is ouer it, and so remayneth 14, as thus.

```
                1 4
Then set I forwarde the  3 8 4 6 8 (1
diuisor, and synde ouer it  x 4
144, as apereth: then seeke I that numbre  1 4
in the table and synde it,
and against it is 6, ther-  3 8 4 6 8 (1
fore I set 6 before 1, for  x 4 4
my quotient, and I take  2
that 144 for the iuste multiplicatiō of the
diuisor by that quotient, and therfore wyth-
out any newe multiplication, I do subtract
that 144 from the other 144, and there re-
steth nothyng, as you maye see.
Therfore I set forwarde  x 4
the diuisor, but leynge it  3 8 4 6 8 (16
will not bee in the nexte  x 4 4
place, (for then ouer  2  x
wold be nothyng) I set it forward twise, as
you see here.  x 4
And for bicause that I  3 8 4 6 8 (16
coulde not sette it in the  x 4 4 2 4
next place folowynge.  x
therfore I sette a cyphre in the quotient, as
you see than loke I ouer the diuisour & finde
68, which I can not find in ỹ table, therfore
take I the next beneth it, which I find in ỹ
table, & that is 48, & agaynste it standeth 2,
whiche I take for the quotiente. And then
without anye multiplying of the quotiente
into
```

into the diuisor, I do subtract that 48 from 68, and there resteth 20, as here appeareth.

```
  A  20
3 8 4 6 8 ( 1602
2 4 4 2 4
  2
```

And so haue I ended the whole diuision. Maist. Now canne you sufficiently skill in these kynds of Arithmetike. And now for the vse of these two last, that is Multiplication and Diuision, I wyll breefly shew you the feate of Reduction.

REDVCTION.

REductiō is, by which all summes of grosse denominatiō may be turned into summes of more subtil denomination: And contrary wayes: all summes of subtyle denominatiō maye bee brought to summes of groser denomination. S. What call you grosse denomination & subtile denomination? M. That I call a grosse denomination, whych dooth contayne vnder it many other subtiller or smaller: as a pound in respect to shyllynges, is a grosse denomination, for it is greater then shyllynges and contayneth many of them. And shyllynges in comparison to

Greſſe denominatiō

pou vnder

poundes, are a subtile denomination, for bi-
cause they ar lesser then poundes, and many
of them are cõtained in one of the other: and
so lyke waies of other thynges what soeuer
thyng is compared to other, if it be greater,
and containeth many of them, it is a grosser
denomination: but if it be lesser, so that ma
ny of theym are in the other, then are they
called subtyle denominations: wherby you
may perceaue, that one denomination may
bee called a grosse denomination, and also a
subtile (that is to say, a great and smalle) in
diuers comparisons. For shillynges compa
red to poundes, are a subtile or small deno-
mination, but compared to penies, they are
a grosse or greate denomination. Sco. Now
I vnderstande the name, I pray you teache
me the vse. M. The vse is easyly lerned, yf
you remembre what you haue learned bee-
fore. For if you wyll reduce any summe of a
grosse denomination into a summe of a smal
ler or subtyler denomination, you must con-
sider how many of that subtiller denomina-
tion do make one of the grosser denomina-
tiõ, and by ye nũbre or numerator do you mul
tiply the other sum: as if you wolde reduce
20 poundes into shillyngs, you must cõsider
that in a pound ar included 20 shilligs, ther
fore multiplye the one 20 by the other 20, &
there will amounte 400, whereby you maye
know, that in 20 poundes are conteined 400

Subtyll d
nominatio

Grosse a
subtyll de
minatio

shillynges. Likewais if you wold reduce 30 shillynges into penies, considering that in 1 shillynge are 12 pennies, you must multiplye 30 by 12, and it will bee 360, whereby you fynd, that in 30 shillynges ar conteined 360 pennies. And thus maie you reduce any grosse denominatiō into a more subtiler, by multiplication, if you know howe many of the lesser dooe make the greater: of whiche thyng I wyll anon gyue you a syxte table for the moste accustomed kyndes of money weightes, measures, and tyme, and suche lyke, whereby you may knowe howe often eche subtile denomination is conteyned in the grosser, whan you shal nede it, for the foresayd kynd of reduction. And also the same shall serue you, if you wold reduce any sūme of a subtiler denomination into a summe of a grosser denomination: For in suche reduction you must cōsider (as in the other forme) how many of the smaller do make the greater, and by that numbre must you diuide the other summe, and the quotient wyll declare how many of the greater denomination ar comprehended in that summe: as for example, If you wolde knowe howe many shillynges are conteined in 3240 pens, consider that 12 pennies do make 1 shilling, you must diuide that 3240 by 12, & your quotient will be 270, whereby you know that so many shillynges are in 3240 pennies. But & you wold

knowe farther, how manny poundes are in thoſe 270 ſhillynges, ſeyng that euery pound conteyneth 20 ſhillynges, diuide that 270 by 20, and it will be 13, and 10 remaynyng, wherby you may knowe, that in 3240 pen nies or 270 ſhillynges, are 13 poundes and 10 ſhillynges. For euermore the remayner muſt be named by ỹ name or denomination of the ſumme that was diuided, whyche in this place were ſhillynges. And thus maye you doo with any other kyndes of denomi nations. Wherefore to the intent that you may haue a lyght knowlege in the common coynes, weyghtes, meſures, and ſuch other, I haue prepared here a breſe table, whiche ſhall ſuffiſe to you at this tyme, tyl hereafter at more conuenient opportunitie I maye in ſtructe you more exactely in the ſame.

A table for Englyſhe coynes.

A Souerayne.	A quarter noble.
Halfe a ſouerayne.	A crowne,
A royall,	Halfe a crowne.
Halfe a royall,	A ſtowne.
A quarter royall.	A grote.
An olde noble,	A harpe grote.
Halfe an olde noble.	A peny or 2 penies.
An angell.	A bandy prattie.
Halfe an angell.	A penny.
A georye noble,	An halfepenny.
Halfe a georye noble	A farthyng.

Engly coynes

The valewe of Englyshe coynes.

value Englyshe coynes.

A Soueraygn is the greatest englyshe coyne, and conteyneth 2 Rotalles or 3 Angelles, eyther 9 halfe crownes, or 4 crownes and an halfe, that is to say, 22 s 6 d.

Halfe a Soueraygn is equall with a Rotall.

A Rotall conteyneth an Aungell and halfe, that is to say, 11 s 3 d.

Halfe a Rotall conteyneth 5 s. 7 d. ob.

A quarter of a Rotall conteyneth 2 shillinges, 9 d. ob. q.

An olde Noble, called an Henry, is woorth 2 crownes, or a noble & halfe, that is 10 s.

Halfe an olde noble is woorth 5 s.

An angell conteyneth a crown and halfe, or 3 halfe crownes, that is 7 s, 6 d.

Halfe an angell is woorth 3 s, 9 d.

A Noble, called a George, is woorth 6 s, 8 d.

Halfe a Noble is woorth 3 s, 4 d.

A quarter of a Noble (which in the olde statutes is called a farthyng, conteyneth 20 d.

A Crowne conteyneth 5 s. and the halfe crown 2. s, 6 d. How be it there is an other crowne of 4 s, 6 d, whyche is knowen by the rose syde, for the rose hath no crowne ouer it, as in the other crowne, but it is enuironed on the 4 quarters with 4 floure deluces, wherby you may best knowe it. But I wyll retourne to speake of the value of the coynes. for I entende not nowe to describe

the

the formes of them. Now of golde are there no more common coynes. In syluer, the greatest is a Grote, which conteyneth 4 penies. Than is there another Grote called an Harpe, which goeth for 3 d. Then nexte is the pennye of 2 d, And then a dandypratte worth thre halfe pennyes. Nexte it a penny, then a halfe penny, and laste and least of al a farthyng, whose coyne is on the one side a crosse, and on the other a purculles. This I tell you, bicause I see many ꝑ can not know a farthynge from a small halfpennye. Now haue I told you al the englysh coines bothe of golde and syluer, but yet of the two most comon valewers of money spake I nothing that is to saye, of poundes and shyllynges, whych though they haue no coynes, yet is there no name more in vse then they, of whi che the shyllings contayneth 12 penies or 3 grotes, & the pound 2 old Nobles, 3 George nobles, or 4 crounes, that is to saye 20 s. And this is now the rate of English mony. Here wolde I now expresse the valewes of sundrye other coynes of diuers countreys, but for three causes I nowe refraine. The first and chiefest is, bycause they are not cur rant by the statutes of this realm. An other cause is, by reason they are so vncertayne, that they be neuer longe at one rate. And a gayne they are so different in so manye pla ces, that it were matter inough for a great

Siluer nes.

boke to speak sufficiently of the all. Howbeit, yet because you shall not be al together ignorant of them, I wyll shew you the valewes of some that are moste in vse: and fyrst of France. The moste common mony are, deniers, soulx, and frankes, 12 deniers make 1 ß, 20 soulx make one franc, so that as you se these 3 kyndes are lyke in the rate to pennies, shyllynges and poundes with vs, but that this is the difference, that their denier is but the 9 part of our pennye, and so their soulx (commonly called sowses) go 9 to our shylling, and 9 of their frances to an englysh pounde of monny: so that 3 of their frankes make a noble. And by those 3 may you practise how to reduce fren he mony into Englyshe mony. And as for the rest of their coynes I wyll omytte tyl another time, when I entende to shew you the rate of sundry other kyndes of mony. But now as for the coynes of Flaunders bee so chaungable, that you must know theym frome tyme to tyme, els you can not reduce them into our mony certaynly, but yet because that you shal haue an example of their mony to exercyse you with all, you shall take those that be most common, as Styuers, both syngle and double, grotes Flamysh, carolus and gylders. A Flemmysh grote is a lyttle aboue 3 farthynges englysh A syngle styuer is 1 d, ob, q, The double styuers 3 d, q. The syluer Carolus syngle 2 s,

q, q, c

q̄, q, c. The double ſiluer Carolus, is 4 d, ob q̄, q. Then is there alſo the Carolus gylden, which is worth 20 ſtyuers. And the Flemiſh noble is worth 3 Carolus gildens, and xii. ſtiuers. But I wyll let them paſſe nowe, exhorting you to practiſe to reduce thoſe kindes into engliſh monye, accordynge as I haue ſet forth here folowyng: 2160 deniers make 240 d, or 20 ß. 3240 denyers, make 360 d, or 30 ß. 8352 deniers, make 928 d, or 3 li, 17 s, 4 d: 2160 ſoulz, make 240 ſhyllynges: and ſo of other the lyke ratt. But if you wyl reduce Flemyſh mony iuſtly, you muſt reduce it fyrſt into the ſmalleſt parte of englyſh monny that is in that coyne, as for exaumple. If you wolde reduce 368, double ſtyuers into englyſh mony, conſidering that a double ſtiuer contayneth 3 d, q̄, you ſhall fyrſt looke how many q̄ be in the double ſtyuer, and you ſhal fynd them 13, therfore multiply the ſumme of the ſtiuers by 13, and than haue you their valew in farthynges, which is 4784. Nowe if you diuide that by 4, then wyl there appeare the numbre of pens, but better it were to dyuide it by 48 (for ſo manye farthynges are in 1 ſhyllynge) and then wyll the quotient declare the ſumme of ſhyllinges. Lykewayſe yf you woulde reduce any ſumme of ſyngle ſtyuers into Engliſhe monye, you muſt multiply the ſame fyrſte by 13, and than haue you for ſumme of

whiche summe if you diuide by 8, then wyll amount the sum of pennies: or yf you diuide it by 96, the summe of shyllynges wyll appere. But thys marke in all diuision, whan ye wo reduce to bryng one denomination into an other, yf there be any remayner after the diuision, that must be named by the denomination of the grosse sume that was diuided: as for exaumple: I would bryng 254 q̄, into pens, therfore I doo diuide that 254 by 4. for so many farthynges make 1 penny, and the quotient is 63, whych is the summe of the pens, & then remayneth yet 2, which are farthynges styll, as you may proue by diuidyng. And this must be marked in al diuisiō namely when it is doone for reduction. And thus much haue I sayd of mony, now wil I shew you in lyke sort the distinction of weyghtes, after the statutes of England, wher the least portion of wayght, is comonly a grain, meanyng a graph of corne or wheat, drye. and gathered out of the myddle of the eare. Of these graynes in tyme passed 32 wayed iuste 1 pennye of Troye. and than was but 20 pennies in an ounce: But now are there 46 pennies in an ounce, so that there are not fully 14 graynes in one penny. But nowe of ounces after Troye rate (which is the standarde of Englande) 12 doomake 1 pounde. But commonlye there is vsed an other wayght, called Haberdepoyse, in

which

in whiche 16 vnces make a pound. Therfore whan you woulde reduce vnces into poundes, you muste consyder whether youre weyghts be Troy waights or haberdepoise, and if it be Troye weight, you muste diuide your vnces by 12 to bryng them to poũdes, but if it be haberdepoyse, you muste dyuide them by 16. Now agayne, there be greater waightes, which are called an hundred, half hundred, and quarterne, and also halfe a quarterne. &c. S. Why? so there maye bee reckned 20 pound, 40 pound, 200 pounde, and suche innumerable. M. All these are numbres of waygbt, but they haue not common waightes made to their rate, as the other haue. And agayne, these that I dydde name, ar not iust in numbre as they seme by their name, for an hundred is not iuste 100, but is 112 pounde. And so the halfe hundred is 56, the quarter 28, and the halfe quarter 14. And this is the common waightes vsed in moste thinges that are solde by waygbte. Howe be it, there are in some thinges other names: as in wolle, 28 pounde is not called a quarterne, but a todde: and the 14 pounde is not named halfe quarterne, but a stone, and the 7 pound, halfe a stone. Other names bycause they differ in many places, and agree in fewe, I let them passe. But a sacke of wolle by the statutes is symytted to be 26 stone. Now in these, though it be solde by the

a Hundr waight.

Wulle weighte

a Todd

Ston

Sack

hundred, and by the stone, in some places, yet the very weights of it are cloues and weies, so that a cloue shoulde containe 7 pounde, and a wey 32 cloues, that is 224 pounde. Howe be it, some statute booke say, that a cloue should be 6 pound: and some saye also, that a weye dothe containe 36 cloues, and that is commonly vsed, for the common wey is of 296 pounde, that is 36 cloues, reckenynge 7 pound to the cloue, and there is 4 pounde ouer weight. And lette this suffices you at this tyme, touchynge weyghtes. Now of wayghtes are made other measures, both for graynes and lyquoure. For a pound in weight, maketh a pynte in measure, so that 8 pounde (or 8 pyntes) doo make a gallon: half a gallon is named a potell: and halfe a potell is called a quart, whyche containeth two pyntes. Now aboue a gallō the nexte measure is a fyrkyn: then a tertian, a kylderkyn or half barrell, and a barrell. And by those measures are solde commonly Ale, beer, wine and oyle, butter and sope, salmon, herrynges and eeles: but as these bee vnlike thynges, so the measure of theim vessels be vnlike: for the measure of ale are as followeth.

Of ale { the fyrken / kilderkē / the barrell } containeth { 8 / 16 / 32 } gallons.

of

Beere measures	the firkyn	conteineth	9	gallons
	the kylderkin		18	
	the barrell		36	

Sope measures, bothe fyrken, kylderkyn and barrell, shulde be all equall to Ale measures. Moreouer the statutes doth lymytte the weight of euery of those three vessels beyng empty.

A barrell	to they empty	26	pound.
Halfe barrel		13	
A fyrkyn		6½	

Hearrynges also be solde by the same measures that ale and sope be solde by: but hearyngs also are sold by the tale, xxiiij. to the hundred tenne thousand to the last.

Hearringe

Salmon and eeles haue a greater measure

Salmõ eeles	the butte	holdeth	84	galons
	½ barrell		42	
	halfe barrell		21	
	the firkin		10½	

Salmon & Eeles,

How be it some statutes dyd lymit eele vessels equall with hearryng vessels.

Nowe as for wyne vessels sydowne are smaller then hogges heddes, whyche are of 63 gallons: euery hoggs heade is two barrelles: yet there are many other wyne vesselles, but of them all see this table, and marke the measures one to an other.

Wine measures,

Of wine and oyle	the rondelet	holdeth	18	Gallons.
	the barrell		31	
	½ hogges hed		63	
	the tertian		84	
	the pype		126	
	the tonne		252	

Tertians. But you shall marke that there be other kindes of tertians, for there be tertyans, (that is to say) thirdles of pipes, of hogges hedes, and of barrels, as well of other thinges as of wyne. Also of malueseys and secke, &c. the halfe tonne is not called a pype, but

Butte rather a butte. And thus muche haue I thought meet to tel you at this time. S. And is thys alwayes true? M. I haue tolde you howe it should be, but how it is I maye not say, how they do differe daily from their iuste measure, the gagers can tell you better then I. But I wyll let thys passe now, and speak brefly of the other mesures. And as of weightes there dyd sprynge the lyquide measures, wherof I spake laste: so of the same spryn-

Drye measures. geth dry measures, as peckes, bushels, quarters, and suche lyke, whereby are measured corne and lyke graynes: also salte, lyme, coales and other lyke. And thys is the order & quantitee of them.

A Pecke. A pecke is the measure of 2 gallons.

Bushell A bushell conteyneth 4 peckes.

Quarter. A quarter holdeth 8 bushelles.

Weye. A waye conteyneth 6 quarters.

Thele

These are the common names and measures, but in diuers places there be diuers sortes. And the bushell in many places is 2 bushelles, but then is the bushell there called a stryke. And in some places half a quarter is called a comoke. But these diuersities are so many to tel you brefely thē al. And again sith they ar agaīst the law & statutes. I coūt thē vnmete to be vsed. But now remaineth yet an other kynde of measure, wherby men mete length and breadthe, and those are an ynche, a foote, and suche other: whose names and length, this table sheweth.

Stryke.
Comoke.

Measures to mete length and breadth.

3 graynes of barley make an ynche. An ynche.
12 ynches make a foote. A foote.
3 foote make a yarde. Yarde.
3 foote and 9 ynches make an elle. Elle.
5 yardes and a halfe make a perche. Perche.
1 perche in breadth and 40 in length, doo make a rodde of lande, which some call a roode, some a yarde lande, and some a farthendele. Rodde. Farthēdele.
2 farthendeles make half an acre of groūd,
4 farthendeles make an acre.

Here mought I tell you many thynges els touchyng measure: And also how to reduce straunge measures to our measures, but because it can not well bee doone without the knowledge of fractions, whiche as yet you haue not learned, I wyll let them passe tyll an other tyme, when I shall instructe you in

Geo

Geometry, wherin I should be enforced eyther to repete the same often agayne. S. But yet for of the partes of tyme, I pray you tell me somewhat. M. You knowe that a naturall daie hath 24 houres, and euery houre hath 60 minutes. It nedeth not to tell you, that 7 daies make a weke, & 4 wekes make a cõmon moneth, & 13 moneths make a yere, lacking 1 daie & certain howres and minutes. But of that I shall instructe you hereafter. And here will I make an ende of Reduction for this tyme, whiche though it be counted no kynde seuerall of Arithmetike, yet your se it is no lesse nedeful to be knowē, nor easier to be don, than of any of the other. S. Mary for it seemeth vnto me muche harder thā any other sort, for it requireth the knowledge of so many thyngs. But now sir when you se tyme, I am redy to learne forth, for as much of reductiō as you haue taught me, I remembre, but & if I do at any time forget, I shall haue recourse to the tables, whyche you haue sette forth for me. M. So do you, for it will not bee remembred without exercise. Now Progression I will begyn.

The partes of tyme.

Day.

Houre.
Weke.
Moneth.
Yeare.

PROGRESSION.

Progressio Arithmeticall.

PRogression is a hiede and quicke addition of diuers summes, proceedyng by some certayne and reasonable order. I meane whayne the distance

staunce of euery 2 numbres is equall to the distance of the second numbre frō the fyrst. S. I vnderstande you not well. M. By an example it wyll be playner, as here: 1,2,3, 4,5,6. here you see the seconde to differe from the fyrste but by 1, and so dothe all the other, one exceede an other by 1 styll to the ende. S. This I perceaue. Ma. And lyke wates here: 4,7,10,13,16,19,22,25. Sco. Yea, they proceede by the difference of 3. M. And so I meane that Progression is an art how to adde all suche numbres together, muche quickelyer then by common Addition, and that by this meanes: Telle how many numbres there ar, and if they be odde, wryte they: summe downe by it selfe: as in this example, 2,4,6,8,10,12,14, where the numbres are 7, as you may see, therfore set downe 7 in a place alone, as I haue doone in the margent here: then adde togyther the fyrste numbre and the laste, as in this example: adde 2 to 14, and that maketh 16, take halfe of it and multiply by the 7, whiche you noted for the noumbre of the places, and the summe that amounteth is the summe of all those figures added togyther, as in this example: 8 multiplied by 7 make 56. & is the sum of al the figures. S. That wil I proue by a other exāple: I wold know how much this sume is, 5,8,11,14,17 20,23,26,29, I tel 9 places, & they ar 9, I

7

note

gressiõ
hmetri

note. Then I put the firste numbre 5, & the laste 29 together, & they make 34, I take the halfe of it, that is 17, and multiplye by 9, & it maketh 153. That you say is the summe of all the numbres. M. So shal you find it if you try it. S. How shal I try it? M. By Additiõ. For if you adde al the parcels togither, you shall se the same summe amount, if you dyd worke well, and that addition trieth all kyndes of progression. S. Then I can adde by Progression, if the nũbres of the parcels be odde. But what if thei be euẽ? as in this example, 1, 2, 3, 4, 5, 6, 7, 8? Maist. When the numbre of the parcels be euen, then note that also as you dyd before, and lykewayes adde the fyrste summe to the last, and by the half of the numbre of the places do you multiplye it, as in your example, the parcels are 8, that note I, then addyng the firste summe to the laste, there amounteth 9; that doone I multiplye by the halfe of parcelles, that is by 4, and it maketh 36, whyche is the summe of the 8 parcelles. But if you wyll take one rule for these bothe, do thus. Multiply the halfe of the one by the other hole, & the summe will amounte all one. For sometyme it chaunceth that the parcels be odde, so that their half can not bee taken, and sometyme it chaunceth the addition of the fyrste numbre and the laste to bee odde, so that the halfe of it can not be taken: but they wyll

neuer

will neuer be bothe odde. S. The I perceiue this, if there be no more longyng to it. M. This is enough for Progressiō Arithmeticall, howbeit there is an other maner of Progression called Geometricall, when the nūbres encrease by a lyke proportion, that is, if the seconde numbre containe the fyrste 2. 3, or 4 tymes, and so forthe, then the thyrde containeth the seconde so many tymes also: and so the fourth the thyrde, and the fyfthe the fourth: wherfore I set those thre exāples here in the fyrst example 3, 6, 12, 24, 48; you se, that euery numbre 1, 3, 9, 27, 81; cōteineth the other, (that 2, 10, 50, 250. goeth next before hym) 2 tymes: and in the seconde example 3 tymes: in the thyrde example 5 tymes: Now if you wyll know how to fynde easyly the summe of any such numbres, doe thus, consyder by what numbre they be multiplied, whether by 2, 3, 4, 5, or any other, and by the same numbre doe you multiply the last summe in the Progression. S. I pray you worke it by this example, 2, 8, 32, 128, 512, 2048, whiche summe I haue increased by the multiplication into 4. M. Then must I multiply the last sūme (whiche is 2048) by 4 also, and it wyll be 8192. Now must I bate from this summe the fyrst numbre of the Progression, which here is 2, then resteth 8190, whiche summe I must diuide by 1 lesse then was the numbre

Geometricall.

ber that I multiplied by. Seing thā I multiplied by 4. I must diuide by 3, so diuidyng 8190 by 3, the quotient will be 2730, whiche is the summe of al the Progression. And now to proue whether you can do the same, I geue you these numbres to adde, 3, 15, 75, 375, 1875, 9375, 46875. &c. I can not well tell by what noumbre this progression dooth increace. May. In any suche doubte doo thus: Diuide the second numbre by the fyrst, & the quotient wyll shew you the numbre that engendreth the progression. Schol. Then is that numbre in this example 5, for so manny tymes is three in 15. M. So is it. Nowe worke as I taught you. S. The last numbre is 46875, whiche I multiply by 5, and it yeldeth 234375. from which I bate the first numbre of the progression, that is 3, & there resteth 234372, whiche I diuide by 4, for that is 1 lesse than 5, & the quotient is 58593, which is the hole sum of the progressiō. M. Now if you remēbre wel this, you haue lerned the art of progressiō, both Arithmetical, & also Geometrical: whiche you maie proue other by subtraction of eche numbre alone frō the sum: & so will ther nothyng remain, other by addyng togither of all ỹ parcels, for so will the same sum amount. And now for the vse of this rule, I wil put forth to you certayne questions, whiche some doo referre them to Addition, but not so nedy

as

as I doo vnto this rule of progression: and some, as vncircumspectely, referre the same to Duplation.

The fyrst question is this.

If I solde vnto you a horse, hauynge 4 shoes, and in euery shoe 6 nayles, with this condition, that you shall paie for the fyrste nayle, 1 ob. for the second 2 ob. for the third 4, and so foorth, doublyng vnto the ende of all the nayles. Nowe I aske you howe muche wouldе the hole price of the horse come vnto. Sco. Fyrst to know the numbre of the nayles, I must multiply 6 by 4, & that maketh 24, then I wyll do thus. I will write the numbre of the nayles euery one in order from 1 to 24, and agaynste eche numbre of nayles the summe of halfepenies duely, as the order of Duplation requyreth, and in this nexte figure folowyng appereth. Questi sellyng horse.

Than do I resort to the rule of progression, wher I consider, that the encreace of this summe proceedeth by multiplicatiō of 2, and therfore I w multiply the last sum by 2 also and it yeldeth 16777216, from which I abate the first numbre, whiche is 1, & then resteth 16777215, whiche I sholde diuide by 1 lesse, than I dyd multiply. But seyng that it is 1, I neде not to diuide it, for 1 (as I haue before said) doth nother multiplye nor diuide: therfor I take that sum 16777215 for the whole summe of the halfe pennies,

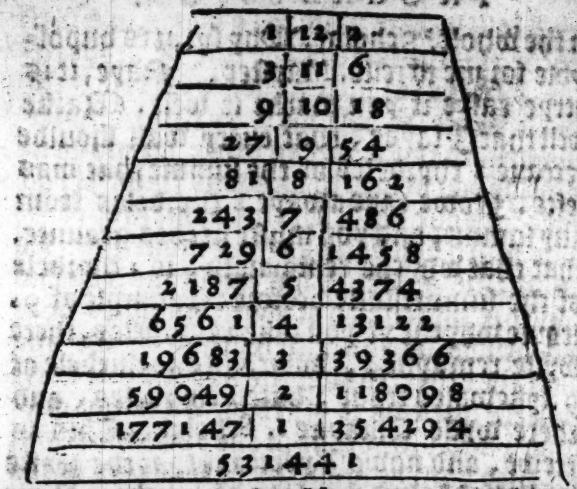
1	1	whiche by reduction I
2	2	fynde to be 69 9050 li,
4	3	and 7, li, ob. that is,
8	4	34952 li, 10 s, 7 d,
16	5	ob. Mayster. That ys
32	6	well done, but I thynk
64	7	you wyll bye no horse
128	8	of the price. Scholar.
256	9	No syr, if I bee wyse.
512	10	Mayster. Well, than
1024	11	aunswere me to thys
2048	12	question.
4096	13	A lorde delyuered to
8192	14	a bryckelayer, a cer-
16384	15	tayne numbre of lodes
32768	16	of brycke, whereof he
65536	17	willed hym to make 12
131072	18	walles, of suche sort,
262144	19	that the fyrste walle
524288	20	shoulde receaue 2 thyr-
1048576	21	deles of the hole num-
2097152	22	bre, and the seconde 2
4194304	23	thyrdeles of that that
8388608	24	was lefte, and so eue-

ry other 2 thyrdeles of that, that remayned. and so dyd the bryckelayer. And when the 12 walles were made, there remayneth one lode of brycke.

Nowe I aske you howe many lode wente to euery walle, and howe manny lode was

in

in the whole? Scholar. Why syr, it is vnpossyble for me to telle. Mayster. Naye, it is verye easye if you marke it well. Marke well that I sayde, that euery walle shoulde receaue 2 thyrdeles of the summe that was lefte. Nowe take awaye 2 thyrdeles from any summe, and you muste needes graunte, that that whyche remayneth, is 1 thyrdele of the summe laste beefore: exaumple of 9. frome whyche yf you take 2 thyrdeles, there wylle remayne 3, whych is one thirdele of 9. lykewaies from 3 bate 2 thyrdeles, and there wylle remayne 1. Scholar. This is trewe, and nowe I perceaue that the leaste walle hadde but two lode of bryche. Mayster. And by the same reason may you knowe howe many lode euery walle hadde, accordyng as this fygure folowyng dooth shew, and lykewayes what the whole summe of bryckes was, for yf you make 12 summes multiplyinge by 3 styll, from the laste remayner, as you see here on the lefte syde of the table, there wyll appeare all the remayners after euery walle: and yf you multyply the laste of those 12 summes by 3 also, than wyl that be the summe of the lodes, whych were deliuered to the brychlayer.



Agayne, yf you do double euerye remainer, as you se at the right side of this table, those numbres wyll shew the summe of lodes that went to eche wall: wherby also you may perceaue, yeche wal was 3 tymes so great as ye next lesser. S. Lo, nowe it appeareth easy ynough. Now surely I see that Arythmetike is a tyght excellent arte. Mai. You wyll say so, whan you knowe more of the vse of it, For thys is nothing in comparison to other pointes that may be wrought by it. Scollar. Thanne I beseeche you syr, ceasse not to instruct me farther in this wonderfull cunning Mayster. By the ordre of the sciences (as men haue taught it) there shuld folow next, the Extraction of rootes of numbre, whyche bycause it is somewhat harde for you, yet

Rule of positiõs

yet I will let te it palle for a whyle, and wyll teache you the feate of the rule of proportions, whyche for his excellency is called the Golden rule. Whose vse is by thre numbres knowen, to fynde out an other vnknowen, whyche you desyre to know, as thus: If you paye for your borde for thre moneths, 16 s, how much shal you pay for 8 moneths? To know this & all such like questions, you shal consider which two of your 3 numbres be of one denomination, and set those 2, the one ouer the other, so that the vndermoste be it, that the question is asked of. As in my question 3 and 8, be bothe of one denomination, for they both be moneths: & bicause 8 is the numbre that the quotient is asked of, I set then one ouer thother, & 8 vndermost, thus.

3 ⌐ with suche a crooked draught of lynes.
8 ⌙ Then doo I set the other numbre, whyche is 16, agaynst 3, at the right syde of the lyne thus, 3 ⌐ 16 And now to know my
8 ⌙ question, this must I doo: I must multyply the lowermost on the left syde, by that on the ryght syde, & the sum that amounteth, I must diuide by the highest on the left syde. Or in plainer wordes thus: I shall multyplye the numbre of whyche the question is asked (whyche ys called the thyrde numbre) by the numbre of an other denomination (whyche is called the second) and that summe that amounteth, muste I

The golden rule.

Question bordyng

The numbre

The s numbre

diuide by the ſumme of lyke denomination, which is called the fyrſt, than for the knowledge of thys queſtion, I multyplye 8, into 16. and there amounteth 128, whiche I dyuide by 3, and it yeldeth 42 s, and 2 s. remayneth, whych I turne into pennies, and they be 24 d. of whych the thyrde parte is 8 d: ſo the thyrd parte of 128 s, is 42 s, 8 d, which ſumme I wryte at the ryght hand of the fygure agaynſt 8, thus. 3 — 16 s

And hereby I knowe, yͤ if 8 — 42 s, 8 d. three monethes boordyng doo come to 16 s, that 8 monethes boordynge wylle come to 42 s, 8 d. and lykewyſe of anye other lyke queſtion. But here muſt you marke, that the fyrſte numbre and the thyrd, be of one denominaciō, and alſo the ſecond and the fourth for whyche you ſeeke: or els bee of ſuch denominacions, that you in woorkynge maye bryng them into one. As if a man ſhuld aſke me thys queſtion: 12 wekes tourneying coſt me 14 nobles, how many poundes is that in one yeare? Here you ſee no 2 numbres of one denommination, but yet in woorkyng you may tourne them into lyke denominations, as thus: Tourne the one yeare into 52 weekes, and the fourthe ſumme wyll bee nobles, by the order of the woorkynge. Then to know this queſtion, multiply the thyrd ſum 52 by the ſecond 14, and the ſumme wyll be 728, that dyuide by 12, and it wyll bee 60,

and

and remaynyng, which yf you tourne into shillynges, they wyll be 53 s, 4 d, whych if it be diuided by 12, will yeld 4 s, 5 d, and ꝑ third part of a penny: put this 60 nobles (which maketh 20 li) with the 4 s, 5 d, and q, and lyttle more: for the summe that answereth to the questiō, and it is the expence of a yeare, and the summes wyll be thus.

12 Z 14 nobles
52 Z 60, 4 s, 5 d, q.

And take this for a generall rule, that euermore the thyrde numbre, be it that the questions ioyned wyth, and the fyrste, the numbre that is of the same denominacion, then must the secōd nedes be that other. And remembre also, that the place of the fyrste numbre is the byggest on the lefte syde: and the place of the seconde rygbte agaynst it on the right syde: the place of the thyrd numbre is vnder the fyrst, as by those examples you haue seene. S. This I rule I can doo. Maist. But and the question be asked thus. In 8 weekes I spend 40 s, how long wyll 105 s serue me? Thoughe the order seeme vnlyke, yet take you 105, for the thyrde numbre, and 40 beynge of the same denominatiō for the first, & then 8 for the second. Then multiply 105 by 8, and it wyl be 840, whych if you diuide by 40, it wyl yelde 21, which is the fourth numbre, and sheweth howe many weekes 105 s. wyll serue, yf you spend 40 s in 8 weekes. The figure of thys

A generall rule.

A generall rule

que-

queſtion is this, as if you ſhuld ſay: yt 40 ſ. ſerue for 8 weekes, 105 ſerue for 21 weekes. Other dyuerſities there be of worckyng by thys rule, but I had leauer that you wolde learne this one well, then at the beginnyng to trouble your mind wyth many formes of worckyng, ſith thys way can do as muche as all the other, and hereafter you ſhall learn the other more conuenyently. But yet before we make an ende of this rule, this ſhall you note, that ther is an other order quite contrarye to this that you haue learned. For in thys rule hytherto, euermore loke, how much the thyrd numbre is greater then the fyrſt, ſo much the fourth numbre is greater then the ſecond: and contrary ways, loke how much the fyrſt ſumme is greater then the thyrde (if it doo chaunce ſo) ſo muche is the ſeconde ſumme greatter then the fourth. But there is a contrarye order, as thys: That the greatter the thyrde ſum is aboue the fyrſte, the leſſer the fourth ſumme is beneth the ſeconde. and this rule you maye call the backer rule: as in exaumple. If I haue bought 30 yardes of clothe, of 2 yardes bredth, and wold bye canuas of 3 yardes broad to lyne it withall, how many yardes ſhuld I neede? S. Why? ther is none ſo brode. M. I doo not care for that. I doo put this example onely for your eaſy vnderſtandyng: For if I ſhuld put the example in other

40 — 8
105 — 21

Bac-
rule.
ſtiō of
ge of
a.

other measures, it wolde be harder to vnder
stande: but now to the matter. If you wold
knowe this question, sette your numbres as
you dyd before: but you shall multiply now
the fyrst numbre by the second, and that a-
riseth therof, you shall dyuide by the thyrd,
whyche thyng if you doo here, I meane, yf
you multyply 30 by 2, it wyll be 60: whyche
summe if you diuide by 3, there wyll appear
20, wherby I know that if 30 yardes of clo-
the of 2 yardes broad shuld be lyned with can
uas of 3 yardes broad, 20 yardes of canuas
wold suffise, as this fygure sheweth.

And now bycause you founde 2 ⟋ 30
fault at my example, how say 3 ⟋ 20
you, perceyue you this? S. Yea syr. I suppose
M. Than answere me to thys question how
many elles of canuas of elle breadthe wyll
serue to lyne 20 yardes of say, of 3 quarters
of a yarde broad? S. In good faythe syr, I
can not tell, for I knowe not how to bryng
the summes to lyke denominacions. M. Thā
I wyll tel you, sith there is mention here of
quarters, & again euery one of ỹ mesure both
elles & yardes may be parted into quarters,
doo you part them so bothe in the breadth &
length, & thā put forth the question by quar-
ters. S. Than I shall say thus, how manye
quarters of canuas of 5 quarters brode, wyl
line 80 quarters of 3 quarters brode? M. now
answer to the questiō. S. First I wil set them

Another like que

downe

bowne in theis fourme thus, for 3 is ioyned wyth the queſtion, and is therefore the thyrde numbre, then is 3 the numbre of the ſame denomination, I mean becauſe they be both referred to bredth. Now I multiply 80 by 3, and it is 240, whiche I deuide by 5, and it yeldeth 48. Then ſay I, that 48 quarters of 5 quarters brode, will ſuffice to lyne 80 quarters of 3 quarters brode. M. Turne the quarters agayne into elles & yardes. S. Then I ſaye, that 9 elles and thre quarters of a yarde of elle broade, will ſerue to lyne 20 yardes of 3 quarters brode, as this figure ſheweth.

3 ⟋ 80
5 ⟋ 48

M. This rule is ſo profitable for all eſtates of menne, that for this rule onely (if there were no more but it) all men were bounde highlye to eſteme Arithmetike. By this rule maye a capitayne in warre worke many thynges, as I wyll hereafter inſtruct you abundantly, onely now I wyll ſhew you this one example. If it ſhulde chaunce a capitaine, whiche hath 4000 ſowldiers, to be ſo encloſed with his enemie, that he coulde haue no freſhe purueiance of vitailes, & that his vitailes which he hath, would ſerue his army but onely 3 moneths, how many me ſhuld he dymiſſe to make the vitaille to ſuffice his reſidue 8 moneths? S. As you taught me, I ſet the numbres thus, ſaying. If 3 moneths ſuffice 4000, to

3 ⟋ 4000
8 ⟋

Queſtiõ of diuiſion wyth out chyng in armye.

how many will 8 moneths suffice? To know this, I multiply the fyrst numbre 3 into the second 40000, & it yeldeth 120000, whiche sume I diuide by 8, & there will be in the quotient 15000, whiche if I do subtracte from 40000, the remainer wyll declare that he must dimisse, 25000, as this figure sheweth.

3 ∠ 40000
8 ∠ 15000

M. Well sythe you perceaue now the vse of this rule, I will shewe other, whiche ensew of the same. And fyrst the Double rule, whych is so called bicause there is in it double worckyng, by whiche thyng onely it differeth frome this. S. Then by an example I shall vnderstande it well enough. M. Soshall you, and let this be the example: If the cariage of 100 poũd weight 30 myles, do coste 12 ß, howe muche will the cariage of 500 weight coste, beyng caryed 100 myles? S. I pray you shewe me the woorkyng of it. M. You muste make 2 worckinges of it: the fyrst thus. If 100 poũd weight coste 12 ß, how much will 500 poũde coste? Set your figure thus, 100 — 12 and multiply 500 by 12, and 500 ∠ therof amounteth 6000, which if you diuide by 100, the quotient will be 60, that is the price of 500 for 30 myles. Then begyn the second worck, saying: If 30 myles coste 60 ß, do we muche will 100 myles coste? Set your figure thus. 30 ∠ 60 And than multiply 100 by 60, 100 ∠

The double rule.

where

whereof amounteth 6000, which beyng diuided by 30, will yelde 200. Than you maie saie that so many pennies shall cost the cartage of 500 pound weight, 100 miles after the rate of 12 d for the 100 caryed 30 myles. S. Now I perceaue it also. M. Then answere me to this question. 30 bushels of wheate sowed, yeldeth in one yere 360, how many will 80 bushels yeld in 7 yere? I meane sowyng euery yere of those 7 syth 80 bushelles. S. First I saie, that if 30 bushelles yelde 360 in one yeare, then 80 bushelles wyll yelde 960 in one yeare: Then for the seconde worke, I say: if one yeare yelde 960, then 7 yeere wyll yelde 6720, as these two figures do shewe

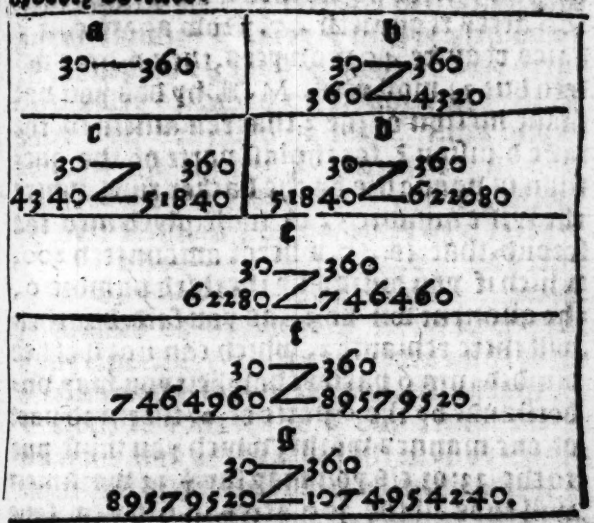
30 — 360 — 1 — 960
80 — 960 — 7 — 6720

But nowe syr, if I set foorth 30 bushels of corne to an other man for 7 yeare agreynge so, that he shal sow euery yere the hole encrese of the corne, and I at the ende of those 7 yeares to haue the halfe of the hole encreale I wolde know how many bushels will there amounte to my part, supposyng the increase to be after the rate of the laste question, for 30 bushelles in one yere 360? M. In suche a question you muste haue so many seuerall worckynges, as there be yeares: as for example. In the fyrst yeare 30 bushels yelde 360, than to know the yeldyng of the second yere I muste say: if 30 yelde 360, howe many yeldeth

wefts of oyng.

estion come,

deth 360? Worke by your rule, and you shall synde 4320. Then say for the thyrde yere: if 30 yelde 360, how many wyll 4320 yelde? you shall haue 51840, and so euery yeare, multiplyeng the whole encrease by 360, and diuidyng it by 30, the encrease of the nexte yeare will amount, as these 7 figures doo orderly declare.



wher I haue set 7 letters for the 7 yeares, of whiche the fyrst is set without arte, because that is the encrease whiche you doue presuppose, and the laste numbre of eche other doeth shewe the encrease of the yere: it standeth for, which 7 letters with declare, so

the

the encreaſe of the 7 yere is 107495424 0 buſhelles, howe many quarters that is, and alſo howe manny waies, you maie by Reduction ſome fynde. Nowe with one queſtion more I ſhall proue you. If 6 mowiers doo mowe 45 acres in 5 daies, howe many mowyers will mowe 300 acres in 6 daies? S. If 45 acres do require 6 mowyers, then 300 acres requyreth 40. Now agayne, if 5 daies require 40 mowyers, then 6 dais needeth but 33 mowyers. M. Why doo you not make mẽcion of the 2 that remaineth in the laſte diuiſion? for the laſt parte of the queſtion is wrought by the backer rule, where the fyrſte numbre 5, is multiplyed into the ſecond, that is 40, whereof amounteth 200, which if you diuide by the third numbre 6, the quotient wil be 33, as you ſaid, but then will there remaine 2, which can not wel bee diuided into 6 partes: howbeit you may vnderſtande by the 6 parte of 2, the thyrd part of one mannes worke, which you muſt put to the 33: or els you may ſay, y 33 workmen will ende all the 300 acres in 6 daies, ſaue two mens worke for one daie, or two daies woorke for one man. But ſuch broken numbre called fractions, you ſhal hereafter more better perceaue, when I ſhall more inſtruct you of them. But nowe will I ſhewe you of the rule of Felowſhyp or Copany, which hath ſundry operations, accordyng to the diuers numbre

eſtio of royage.

le of fe owſhyp.

numbre of the company. This rule is some-
tyme without difference of tyme, and some-
tymes there is in it difference of tyme. First
I wyll speake of that without difference of
tyme, or why the lette this bee an example.
Foure marchaunts of one company made a
banke of mony diuerlly, for the tyrst layd in
30 li. the seconde 50 li. the thirde 60 li. and
the fourth 100 li. which stocke they occupi-
ed so long, tyll it was encreased to 3000 li.
Now I demande of you, what shulde eche
man receiue at the partyng of this money?
S. I perceiue that this rule is lyke the other,
but yet ther is a difference which I perceiue
not. M. Then will I shew it to you. First by
addition, you shal bring al the particular sum-
mes of the marchaunts into one summe,
which shall be the first summe in your wor-
kyng by the golden rule, and the whole sum
of the gaynes with the stocke shalbe the se-
cond summe. Now for the third summe, you
shal set the portion of eche man one after an
other, and then worke by the golden rule. &
the fourth summe wyll shew you eche mans
gaynes: as in exaple. The parcels of thole
foure marchaunts make in one summe 240
li, set that in the tyrste place, the gaynes in
the seconde, and the tyrste mannes portion
of stocke in the thyrde place, thus.

Without ryme.

Question of a banke.

240 — 3000
30 — ?

Now multiplye the seconde
by the thyrde, and it wyll be

720000, whiche you shall diuide by 240, and there will appere 375 li. thus,

240 — 3000 and that is the gayns for the
30 — 375 first man. Now for the secõd ma set the 50 li. that he brought in the third place, and worke as before. and his part will be 625 pounde, as this figure sheweth.

240 — 3000 Like wise for the third man
50 — 625 set his mony, which was 60 li, and his part of gaynes will be 750 li. as here appeteth.

240 — 3000
And so for the foureth man, 60 — 750
if you set his summe, whiche is 100 li. his
240 — 3000 gains wil be 1250 li. as the proue wil declare
100 — 1250 S. This I perceaue. but is ther any way to examin whether I haue well done or no? M. That must you do by one comon proue, whiche serueth to the golden rule, and all other enusyng of the same: and that is this. Chãge the standinges of the numbres, and set the third in the first place, the fourth in the secõd place, and the first in the third place, & thã worke by the golden rule, and if you haue done wel, the fourth nũbre now will be the same, that was the second before. As for exãple, I will take the laste worke which was this.

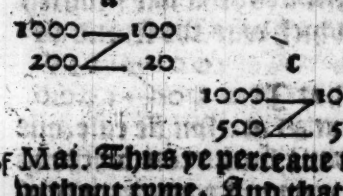
240 — 3000 which to examin, I alter as
100 — 1250 I sayd thus. 100 — 1250
Now if I multiply the se- 240 —
conde numbre by the third, and diuide that

proofe.

that

that amounteth by the firſt, thā will ꝑ fourth numbre be 1000, which was the ſecond before as you ſee here: 100 — 1250
which is a token that I 240 — 3000
haue wel doone But as in a ſyngle rule, one proue thus is ſufficient, ſo in a rule wher many operations be, you muſt tourne euery of them as I haue done with this one. Scoler. Then for the proue of the firſt worke of this rule, I ſhoulde tourne the numbres thus,
30 — 175 And for the 50 — 625
240 — ſecond thus. 240 —
And for the thirde thus.
60 — 750 And in eche of theym if the
240 — worke wer true, ꝑ fourth numbre will be ſtill 3000. Mai. Well, nowe an other example will I put to you, not of gaines, but of loſſe: for one reaſon ſerueth for both. If thre marchantes in one ſhippe and of one felowſhyp, had bought marchandiſe, ſo ꝑ the firſt had layd out 200 pound, the ſecond 300 li. and the thyrd 500 pounde, & it chaunced by tempeſt that they dyd caſt ouerboard into the ſea marchādiſe of the value of 400 pound, how much ſhuld eche man beare in this loſſe? S. If I ſhall doo in this as you dyd in the other queſtion, than muſt I ioyn their thre portions togither, 200, 300, & 500 whiche maketh 1000: than ſaye I, if 1000 loſe 400, than ſhall 200 loſe 80, and 300 ſhall loſe 120, and 500 ſhall loſe 200, as by theſe thre figures it dooth appere playne. Queſtion of loſſe.

M.ii 1200.



e rule of woshyp o tyme.

Mai. Thus ye perceaue the vse of the Rule without tyme. And that you maye as well perceaue the same with diuersitie of tyme, I propose this exãple: Four marchants made a common stocke, whyche at the yeres ende was encreased to 35145 li. Now to knowe what shall be eche mans portion of gaynes, you must knowe eche mans stocke and tyme of continuance: The first man of these foure layde in 669 li, which he dyd take from the stocke agayne at the ende of 10 monethes: The seconde man layde in 810 li. for 8 monethes. The third layd in 900 poũd, for 7 moneths. And the fourth laid in 1040 poũds for 12 monethes. This question shall you examin as you dyd the other before, sauyng that where as in the thyrde place of the figure you set eche mans summe alone, here you shall set the same beeynge multiplied by the numbre of their tyme: and likewaies in the first place of the figure, you shal set their hoole summes so multiplied by their tyme, and added into one summe, as thus: The first mans summe 669 poũd, which I multiply by 10 (that was the nũbre of his tyme) and it maketh 6690. The seconde mannes summe 810 poũd, multiplied by 8 (which was

estiõ of anke.

his tyme) maketh 6480. The thirde mans summe 900 li. multiplied by 7 (for that was his tyme) yeldeth 6300. The fourthe mans summe was 1040 li. and his tyme 12, multiply the one by the other, & it wil be 12480. Theſe iiij. ſummes thus multiplied by their tyme, muſt be ſet orderly in the thyrde place of the figure, & in the firſt place muſte be ſet the holle ſumme of all. iiij. which is 31950. Now to end the queſtiõ, I ſay firſt, if 31950 did get 35145, what did 6690 get? Anſwer 7359 li. as by this figure appereth.

8

31950 — 35145
6690 — 7359

Like waies the ſecõd mã had to his part 7128 li. the third muſt haue 6930 li. And the iiij. man ſhall haue for his parte 13728 li. as theſe figures do partelly declare

b

31950 — 35145
6480 — 7128

c

31950 — 35145
6300 — 6930

d

31950 — 35145
12480 — 13728

S. This I lyke very well: but what proſe is there of this worke? Map. The ſame that I taught you for the other. Howbeit there is vſed both for this worke & the other alſo this maner of proſe, to adde all the partes togither, & if they agree to the holle ſum, than ſemeth it wel done: but this is no ſure rule. S. yet wil I proue it in this exãple. The foure

Proofe.

M iij parcels

parcels ar these, which if I adde to- 7359
gither, there wyl amount 35145, & 7128
that was the whole sum, so is this 6930
rule true here. M. And so wyll it be 1377 28
stil when ỹ worke is truely done. But yf you
list to see it proued false, take 10000 fr. from
the fourth man, and put it to anye of the o-
ther 3, & than be you sure that you haue not
don wel, & yet wyll that proue alowe it, for the
addition will styl be all one. S. It must needs
be so: but what haue I nowe to learne? Ma.
There are many other excellēt parts behind
of whych I wyll not as nowe make mention
bicause that without the knowledg of fracti-
ons they can not be duely taught, and much
lesse vnderstand: Therfore will I propose to
you iii. or iiii. questions more, whereby you
may practise the better the feat of the rule of
felowshyp, and so make an end for this time.
But this may not be forgotten, that in al su-
che questions if the monny be of dyuers kin-
des, you must by reduction bryng it to one
kynde, that is to say, to the least valure that
is named in ỹ question. And lykewyse shall
you doo, yf the tyme be of dyuers kindes: as
some yeres, some moneths, weekes & daies:
you shall make al monethes, weekes or daies
accordinge as the least name of tyme in the
question is: As for exāple. First in diuersitie
of mony, iii cōpanions bought 2000 sheepe
& paid for them 241 pound, 13 s. 4 d, of which

Reduction of coyne.

summe

summe one paid 101 li. 10 s, The seco̅d payd 52 li, 17 s 10 d. And the third payd 57 li. 5, s, 6 d. Howe many sheep must eche of theym haue? Answer: The first shal haue 840. The second 686. And the third 474. And that must you worke thus: Fyrst considering that your money is of dyuers denominacio̅s, you shall by reduction bryng it all into the smalest denominacion which is in it. that is to say, pens, and so wil the total summe be 58000 d. Now if you turne eche mans money into pennies also, the first mannes summe wil bee 24360 pens. The second mans summe 19894 d. And the thyrde mannes money wyll bee 13746 d. Nowe to knowe howe many sheepe euery man shall haue, serue the whole summe of money, that is 58000 d, in the fyrst place, and in the second place set the numbre of sheep, and then ordrely in the thyrd place sette eche mannes money: and then multiplying the third and second summes togyther, and diuydyng that that amounteth by the fyrst. there wyll appear the numbre of sheepe that eche ma̅ ought to haue as these thre figures doo shewe.

a b

58000 ⁄ 2000 58900 ⁄ 2000

24360 ⁄ 840 19894 ⁄ 686

c

58000 ⁄ 2000

13946 ⁄ 474

Scolar. Why doo you sette the mony in the first place, seing in the questiō you say, 2000 shepe coste 58000 ō, and not thus, 58000 ō cost 2000 shepe? Ma. And you remembre, I taught you at the beginning of this golden rule, that the fyrst and third numbre must be of one name, and of like thynges, and euermore the numbre that the question is asked of, must be set in the thyrd place. Now is the question plainely this. If 4 men boughte 2000 shepe for 58000 ō, how manye shepe shall eche man haue? But seyng in this question there ought more respect to bee had to the summe of money, than to the summe of the persons (for in summes of monye is there proportion toward the shepe, and not in the numbre of persons) therfore must wee turne the question, thus: If 58000 ō bought 2000 shepe, how many did 24360 ō bye? Agayne, how manye dyd 19894 ō bye? and how many bought 13746 ō? S. I perceaue it reasonable, and so shall I doo in all lyke questions. M. Euen so. But for easynes of the woorke, marke this, when so euer the first & second numbre hath cyphers in their fyrste places, you may both in the multiplycation and in the diuision leaue out those cyphers, so that you leaue out lyke many out of both summes, as in this question, the fyrste noumbre 58000 hath three cyphres, and so hathe the seconde, that is 2000, ther-

therefore take awaye theyr cyphers, and ſo wyll the fyrſt numbre be 58, and the ſecond 2, ſet them in theire places, and worke according to the rule, and you ſhall perceiue that it wyll be all one, ſauing that this is the ſhorter and eaſier way, as theſe thre fygures do ſhowe.

a

58 ⁄ 2
24360 ⁄ 840

b

58 ⁄ 2
19894 ⁄ 686

c

58 ⁄ 2
13746 ⁄ 474

And thys you ſee is both the eaſyer and alſo the more certaine way, to know the anſwere to thys queſtion. S. Truth it is as you ſaye: But fyr, me ſeemeth I myght aſke a farther queſtion heere, not onelye how many ſheepe eche man ſhuld haue, but alſo what euerye ſhepe coſte. M. That queſtion doth not onely belonge to this rule, but may alſo be diſcuſſed by Diuiſion, eſpecially if the queſtiõs numbre be one onely, as thus: Dyuide the total ſumme 58000 ℔ by 2000 (eche ſhepe 58 by 2, omytting the cyphres) and the quotiente wyll be 29 ℔. that is 29 ℔ 5 ℥: howbeit, by thys rule you may doo it, and beſt whan the numbre of the queſtion doth exceed 1: as yf I ſhuld aſke this queſtion, 2000 ſheepe coſte 58000 ℔, how muche dydde 20 coſte? Then ſhall I ſette my fygure thus,

2000

2000 — 58000 And doyng after the rule
20 — ther wyll amount 580 li
(that is 2 li. 8 s. 4 d) the prce of one scoze.
But if you wyl vse that eaſy way that I dyd
teache you, you may chaunge the fyrſt and
ſecond numbze, thus. 2 — 58
Yet nowe one queſtion moze wyll 20 —
I moue (ſ you may perceaue the vſe of al o-
ther lyke) ſo make an end. There is in a ca
thedzal thurch 20 cannones, and 30 vicars.
Queſtion. thoſe may ſpend by yeare. 2600 li. but euery
cannon muſt haue to his parte, 5 tymes ſo
much as euery vicar hathe, howe muche is
euery mans poztion ſay you? S. I pray you
make the anſwere your ſelfe, ſo ſhall I per-
ceaue beſt the meanes to anſwere to ſuch o-
therlyke. M. In thys queſtion you muſt do
as in thoſe that haue diuerſite of tyme, foz
here is dyuerſity of pozcions: Therfoze ſhall
you multiplye the numbze of the perſons by
their difference of pozcion (as you dyd in the
other by tyme) then muſte you multiplye the
20 (whyche is the numbze of canonnes) by
5, foz that is the numbze of theyz pozti n, ſo
will it be 100: then 30 (that is the numbze of
vicars) by 1 that is the numbze of theyz poz-
tion, and it wyll be 30: put thoſe two ſum-
mes togither, and they make 130, than ſaye
thus: If 130 ſpend 2600 li. what may 100
ſpende? The rule ſhoweth 2000 li. Agayne
foz the vicares: If 130 ſpende 2600 pound,
what

what maye 30 spend? Aunswere, 600 li, as these figures shew.

130 — 2600 | 130 — 2600
100 — 2000 | 30 — 600

But yf euery Cannon shuld haue so oftentymes 4 li, as the vicar shulde haue 3 li, than shold I multiply 20 by 4, (that were 80) and 30 by 3 (that were 90) and than bothe were 170. Then should the figures be sette thus;

| li, s. d. | li, s. d.
170 — 2600 | 170 — 2600
80 — 1223,10,7 | 90 — 1376,9,5

But this sorte is to harde for you by reason of the fractions, therfore I wyll let it rest to that place. And by this rule you se what the twenty cannons may spend: whych summe if you deuide by twenty, you shal se eche cannons portion, and so of the tyears, if you deuyde their summe by thyrtye, the quotient wyll declare euery tyears portion.

The

2000 — 58000 And doing after the rule
20 — ther wyll amount 580 li
(that is 2 li, 8 s. 4 d) the price of one score.
But if you wyl vse that easy way that I dyd
teache you, you may chaunge the fyrst and
second numbre, thus. 2 — 58
Yet nowe one question more wyll 20 —
I moue (y you may perceaue the vse of al o-
ther lyke) & so make an end. There is in a ca
thedral church 20 cannones, and 30 vicars.
Question. those may spend by yeare. 2600 li. but euery
cannon must haue to his parte, 5 tymes so
much as euery vicar hathe, howe muche is
euery mans portion say you? S. I pray you
make the answere your selfe, so shall I per-
ceaue best the meanes to answere to such o-
therlyke. M. In thys question you must do
as in those that haue diuersitie of tyme, for
here is dyuersitie of portions: Therfore shall
you multiplye the numbre of the persons by
their difference of portion (as you dyd in the
other by tyme) then muste you multiplye the
20 (whyche is the numbre of canonnes) by
5, for that is the numbre of theyr portion, so
will it be 100: then 30 (that is the numbre of
vicars) by 1 that is the numbre of theyr por-
tion, and it wyll be 30: put those twoo sum-
mes together, and they make 130, than saye
thus: If 130 spend 2600 li. what may 100
spende? The rule showeth 2000 li. Agayne
for the vicares: If 130 spende 2600 pound,
what

what maye 30 spende? Answere, 600 li., as these figures shew.

130 ⟋ 2600 | 130 ⟋ 2600
100 ⟍ 2000 | 30 ⟍ 600

But yf euery Cannon shulde haue so oftentymes 4 li. as the vicar shulde haue 3 li., than shold I multeply 20 by 4, (that were 80) and 30 by 3 (that were 90) and than bothe were 170. Then should the figures be sette thus;

| li, s, d. | li, s, d
---|---|---
170 ⟋ 2600 | 170 ⟋ 2600
80 ⟍ 1223,10,7 | 90 ⟍ 1376,9,5

But this sorte is to harde for you by reason of the fractions, therfore I wyll let it rest to that place. And by this rule you se what the twenty cannons may spende: whych summe if you deuide by twenty, you shal se eche cannons portion, and so of the yeares, if you deuyde their summe by thyrtye, the quotient wyll declare euery yeares portion.

The

¶ The seconde dyalogue of accomptyng by counters.

MAYSTER.

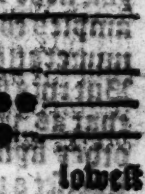


Owe that you haue learned the commen kyndes of Arithmetike wyth the penne, you shal see the same arte in coũters: why the feate dooth not onely serue for them that can not wryte ne read, but also for the that can do both, but haue not at some tymes their penne or tables ready wyth them. This sorte is in two formes cõmonly, the one by lynes, and the other wythout lynes. In that that hath lines, the lines do stãd for the ordre of places. and in that that hath no lynes, there muste be sette in their steede so many counters as shall neede, for eche lyne one, and they shall supply the stede of the lynes. S. By examples I shuld better perceiue your meanyng. M. For example of the lines lo, here you see vi. lynes, which stande for syxe places, so that the nethermost standeth for the first place, and the next

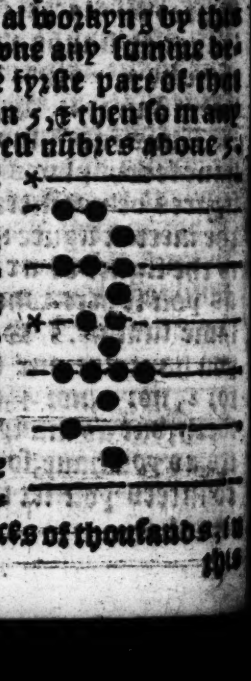
—100000—
—10000—
—1000—
—100—
—10—
—1—

about

aboue it, for the seconde: and so vpward tyl you come to the highest, whiche is the fyrste lyne, and standeth for the fyrste place. Now what is the valewe of euery place or lyne, you may perceaue by the figures whyche I haue set on them, which is according as you learned before in the Numeratiō of figures by the penne: for the fyrst place is the place of vnitees or ones, and euery counter set in that lyne, betokeneth but one: and the secōd lyne in the place of .10, for euery coūter there standeth for 10. The third lyne, the place of hundredes, the fourth of thousandes, and so forth. S. Syr I doo perceaue that the same order is here of lynes, as was in the other figures by places, so that you shall not nede longer to stande about Numeration, except there be any other difference. M. If you do vnderstande it, then how wyl you set 1543? S. Thus, as I suppose. M. You haue set the places truly, but your figures be not mete for this vse: for the meetest figure in this behalfe, is the figure of a counter rounde, as you se here. where I haue expressed that same summe. S. So that you haue not one figure for 2, nor 3, nor 4, and so forth, but as many diget its as you haue, so many counters you set in the lowest



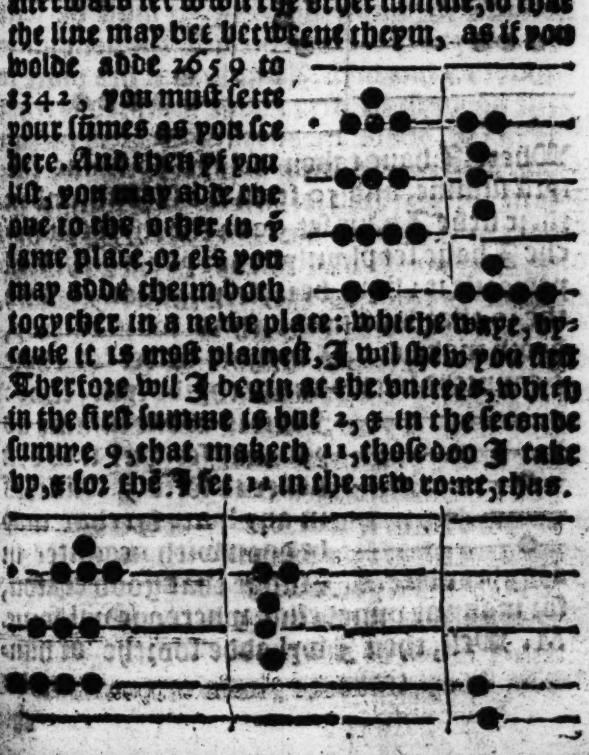
lowest lyne: and for euery 10 you set one in the second lyne, & so of other. But I know not by what reason you set that one counter for 500 betweene twō lines. M. You shall remembre this, that when soeuer you neede to set downe 5, 50, or 500, or 5000, or so forthe any other numbre, whose numeratour is 5, you shall sette one counter for it in the next space aboue the lyne, that it hath his denomination of, as in this example of that 500, because the numerator is 5, it must be sette in a voyde space: and because the denominatour is a hundred, I know that his place is the voyde space next aboue hundredes, that is to say, aboue the thyrd lyne. And farther you shall marke, that in al workyng by this forte, yf you shall set downe any summe betweene 4 and 10 for the fyrste part of that numbre you shal set down 5, & then so many counters more, as ther rest vnbres aboue 5. And this is true both of digetts & articles. And for exãple I wil set down this summ 187965, whiche sum if you mark wel you neede none other examples for to learne the numeratiō of this forme But this shal you marke that as you bidde in the other kynde of arithmetike, set a prick in the places of thousands, in



this

this worke you shal sette a starre, as you see here. S. Then I perceaue numeration: but I pray you, howe shall I doo in this art to adde two summes or more together? M. The easiest waie in this art, is to adde but 2 summes at ons together, howbeit, you may adde more as I will tell you anon. therfore when you wil adde two summes, you shall first set down one of them, it forceth not which: & then draw a line crosse the other lines. And afterward set down the other summe, so that the line may bee betweene theym, as if you wolde adde 2659 to 8342, you must sette your summes as you see here. And then yf you list, you may adde the one to the other in the same place, or els you may adde theim both together in a newe place: whiche waye, because it is most plainest, I wil shew you first. Therfore wil I begin at the vnittes, which in the first summe is but 2, & in the seconde summe 9, that maketh 11, those doo I take vp, & for the I set 1, in the new rome, thus.

Addition



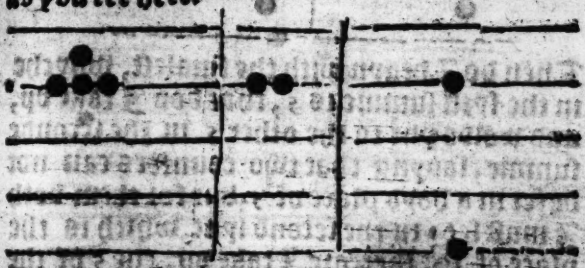
Then doo I take vp all the articles vnder a hundred, which in the fyrſt ſumme are 40, and in the ſeconde ſumme 50, that maketh 90: or you may ſay better, that in the fyrſt ſumme there are 4 articles of 10, and in the ſeconde ſumme 5, whiche make 9, but then take hede that you ſet theim in their tyghte lynes, as you ſee here.



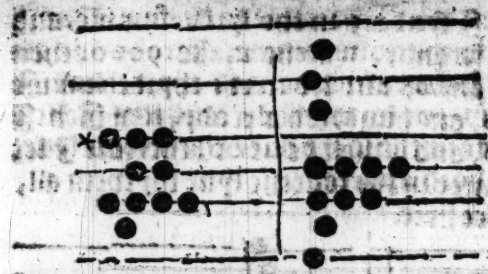
Where I haue taken awaye 40 frome the fyrſt ſumme, and 50 from the ſecond, and in their ſtede I haue ſet 90 in the thyrde, whyche I haue ſet plainly, that you myghte well perceaue it: how be it, keepyng that 90 with the 10 that was in the third roume alredy, dooth make 100, I myghte better for thoſe 6 counters ſette 1 in the thyrde lyne, thus: For it is al one ſumme as you may ſee, but it is beſt neuer to ſette 5 counters in any lyne, for that may be doon with 1 counter in a higher place. S. I iudge that good reaſon, for many ar vnnedefull where one wil ſerue.

M. Well, then I wyl adde foorthe of hundreds

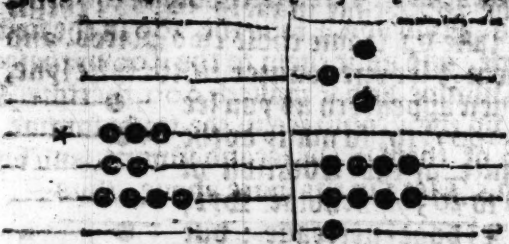
diedes: I fynde 3 in the fyrſte ſumme, and 6 in the ſeconde, whiche make 9000, then doo I take vp and ſet in the thyrde ſumme where is one hundreth already, to which I put 900, and it wyll be 1000, therfore I ſet one counter in the loweſte lyne for them all, as you ſee here.



Then adde I the thouſandes together, whiche in the fyrſt ſumme are 8000, and in the ſeconde 2000, that maketh 10000: then doo I take vp frome thoſe two places, and for them I ſet one counter in the fyrſte lyne, and then appeareth as you ſee to be 11001, for ſo many dothe amounte of the Addition of 1342 to 2659. Scholar. Syr, this I dooe perceaue: but howe ſhall I ſette one ſumme to an other, not chaungyng theym to a thyrde place? Maiſter. Marke well howe I dooe it: I wyll addde together 6543 6, and 32453, whyche fyrſte I ſette downe thus.

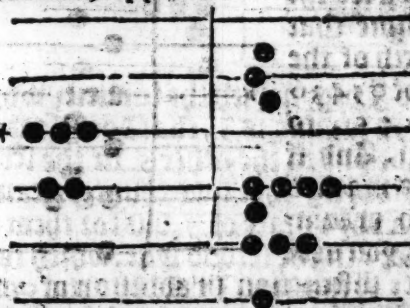


Then do I begyn with the ſmaleſt, whyche in the fyrſt ſumme is 5, that doo I take vp, and wolde put to the other 5 in the ſeconde ſumme, ſayyng that two counters can not be ſet in a voyd place of 5, but for them both I muſt ſet 1 in the ſecond lyne, whych is the place of 10, therfore I take vp the 5 of the fyrſt ſum, and the 5 of the ſecond, & for them I ſet 1 in the ſeconde lyne, as you ſe here.

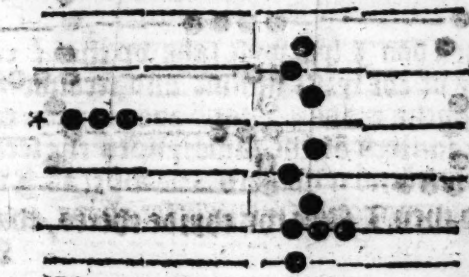


Then doo I lykewyſe take vp the 4 counters of the fyrſte ſumme and ſeconde lyne, (whyche make 40) and adde theym to the 4 counters of the ſame lyne in the ſeconde ſumme, and it maketh 80. But (as I ſayd) I may not conuenientely ſet aboue 4 counters

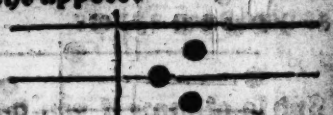
ters in one lyne, therfore to those 4 that I take vp in the fyrste summe, I take one also of the seconde summe, and than haue I taken vp 50, for whiche 5 counters I sette down one in the space ouer the second lyne, as here doth appere.



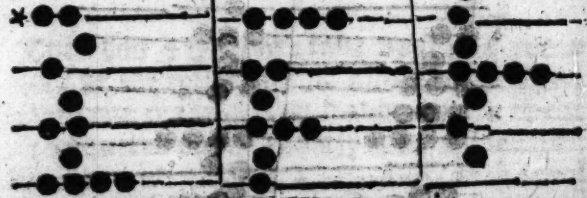
And than is there 80, as well with those 4 counters, as if I had sette downe the other 4 also. Now doo I take the 200 in the firste summe, and adde them to the 400 in the seconde summe, and it maketh 600, therfore I take vp the 2 counters in the fyrste summe, and 3 of them in the second summe, and for them 5 I set 1 in the space aboue, thus.



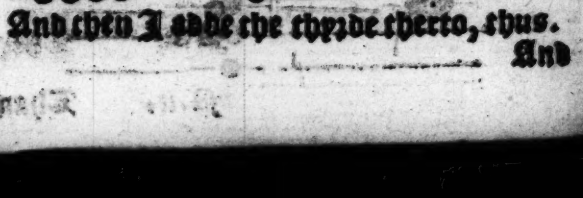
Then I take the 3000 in the fyrſt ſum, vnto which there are none in the ſecõd ſum, ſo geeyng, therfore I doo onely remoue thoſe 3 counters from the fyrſt ſumme into the ſecoude, as here doothe appere.



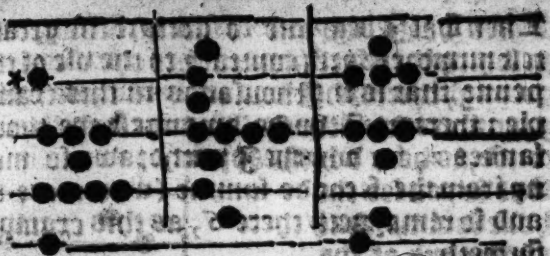
And ſo you ſee the holle ſumme that amounteth of the additiõ of 65436 with 3245, to be 68681. And if you haue marked theſe two examples wel, you nede no further inſtruction in addition of 2 only ſummes: but if you haue more then two ſummes to adde, you maye adde theym thus. Fyrſt adde two of them, and than adde the thyrde and the fourth, or more, yf there be ſo many: as if I would adde 2679, with 4286 and 1791. Fyrſt I adde the two fyrſt ſummes thus.



And then I adde the thyrde therto, thus.



And

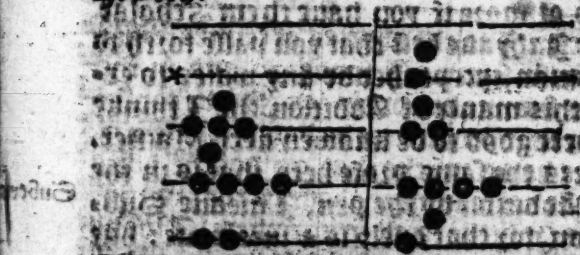


And so of more if you haue them. Scholar Nowe I thynke best that you passe forth to subtraction, except ther be any wates to examyn this maner of Addition, the I thinke that were good to be knowen nere. Maister. There is the same profe here that is in the other Addition by the pen, I meane Subtraction, for that onely is a sure waie: but consyderyng that Subtraction must be first knowen, I wyll fyrst teache you the arte of Subtraction, and that by this example. I woulde subtracte 289 oute of 875. These summes must I set downe, as I did in Addition: but here it is best to set the lesser numbre fyrst, thus. Subtra-



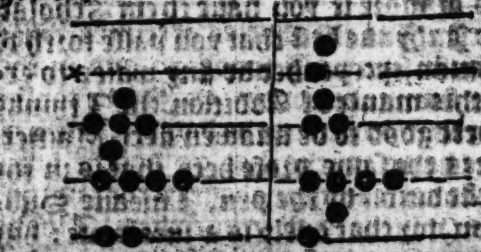
Then

Then shal I begynne to subtract the greatest numbres fyrst (contrary to the vse of the penne) that is the thousandes in this example: therfore I fynde amongest the thousandes 2, for whyche I withdrawe so many from the seconde summe (where are 8) and so remayneth there 6, as this example sheweth.



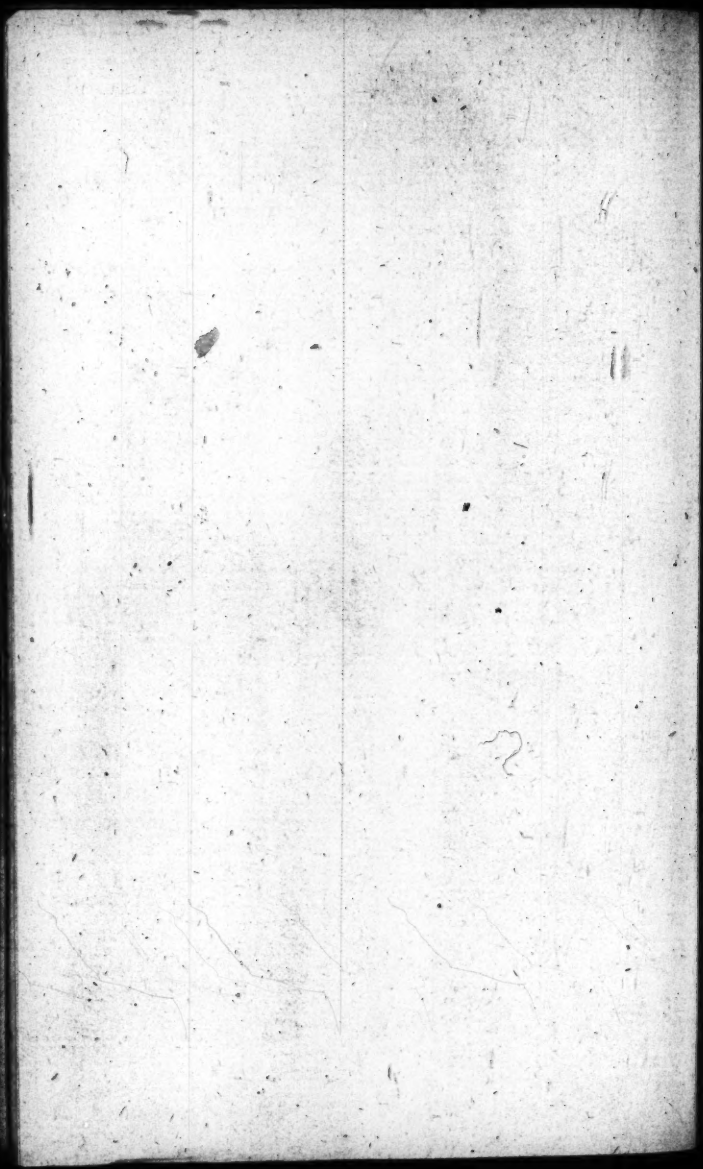
Than do I ieweares with the hundredes, of whyche in the fyrste summe I haue 8, and in the seconde summe but 7, out of whyche I can not take 8, therfore this muste I doo: I must looke howe muche my summe differeth from 10, whyche I fynd here to be 2, then must I bate for my summe of 800, one thousande, and sette downe the excesse of hundredes, that is to say 2, for so much 1000 is more than I shoulde take vp. Therfore from the fyrst summe I take that 800, and frome the seconde summe where are 6000, I take vp one thousande, and leaue 5000: but then sette I downe the 200 vnto the 700, that are there alreadye, to make them

Then shal I begynne to subtract the greatest numbres fyrst (contrary to the vse of the penne) that is the thousandes in this example: therfore I fynde amongest the thousandes 2, for whyche I withdrawe so many from the seconde summe (where are 8) and so remayneth there 6, as this example sheweth.

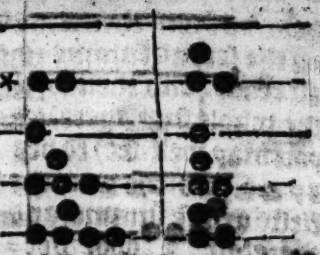


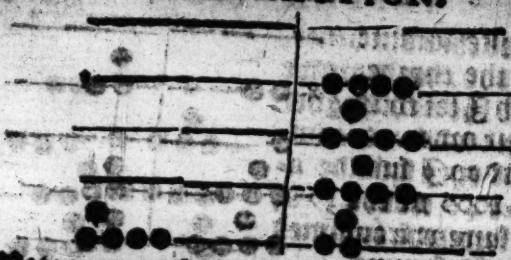
Than do I likewaies with the hundredes, of whyche in the fyrste summe I fynde 8, and in the seconde summe but 7, out of whyche I cannot take 8, therfore this must I doo: I must looke howe muche my summe differeth from 8, whyche I fynde here to be 2, then must I bate for my summe of 800, one thousande, and sette downe the figure of hundredes, that is to say 2, for so much 1000 is more than I shoulde take vp. Therfore from the fyrst summe I take that 800, and from the seconde summe where are 6000, I take vp one thousande, and leaue 5000: but then sette I downe the 200 vnto the 700, that are there alreadye, to make

them

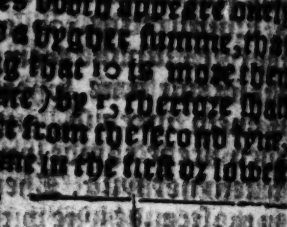


the fyrſte ſumme from the thyrde, whych I ſet thus, in their order. Then doo I ſubtract 2000 of the fyrſt ſumme from the ſecõd ſumme, and that remayneth there 5000 thus: Then in the thyrd lyne, I ſubtracte the 100 of the fyrſte ſumme, from the ſecand ſumme, where is onely 100 alſo, and then in the thyrde lyne reſteth nothyng. Then in the ſeconde lyne with his place ouer hym, I fynde 80, whyche I ſhoulde ſubtracte frome the other ſumme, then ſeeyng there are but onely 70, I muſt take it out of ſome hyguer ſumme, whyche is here onely 5000, therefore I take vp 5000, and ſeeyng that it is ſo muche by 4930, I ſet downe ſo many in the ſeconde roume, whyche with the 70, beyng there alreadye, doo make 5000, and then the ſumme
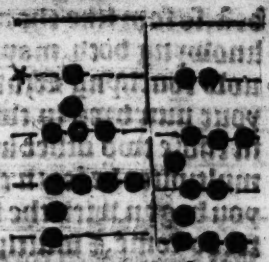




Yet remayneth there in the fyrste ſumme 9 to be baſed from the ſecond ſumme, where in that place of vnities booth appeare onely 7: then I muſte haue a hygher ſumme, that is to ſaye 10, but ſeyng that 10 is more then 9 (whych I ſhuld abate) by 1, therfore I take vp one counter from the ſecond fyre, and ſet downe the ſame in the fyrſt or lower lyne, as you ſee here.



And ſo haue I ended this worke, and the ſumme apereth to be the ſame, whych was the ſecond ſumme of my addition, & therfore I pereaue I haue well done. M. To ſtande longer about this it is but folye, except that you muſte alſo vnderſtand, that many do begyn to ſubtract with counters, not at the hygher ſumme, as I haue taught you, but at the nethermoſte, as they doo vſe to adde: & when the ſumme to be abated in any lyne apereth

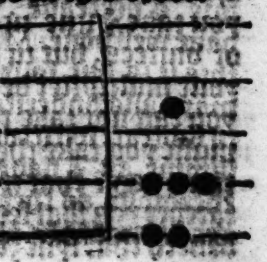
greater

greater then the other, then doo they borowe one of the nexte hygher roume: as for exampple: If they shoulde abate 846 from 2378, they serte the summes thus.



And tyrst they take 6 whiche is in the lower lyne, and his spare from 8 in the same roumes, in the seconde sume, and yet there remaineth 2 counters in the lowest lyne. Than the seconde line must 4 be subtracte frō 7, & so remaineth there 3. That is in the thyrd lyne. and his spare frō 3 of the seconde sume is not bigger therfore as they bate it from a hygher roume, that is, from 1000, because that 100 is tomuche by 100 therfore must I set down 100 in the thyrd lyne, after I haue taken by 1000 from the fourth lyne, than is there yet 1000 in the fourth lyne of the tyrst summe, whiche if I withdrawe frome the seconde summe, than bothe all the figures stand in this order.

So that (as you see) it differeth not gretly whether you begynne Subtraction at the hygher lynes or at the lower. How be it, as commune men lyke the one wape

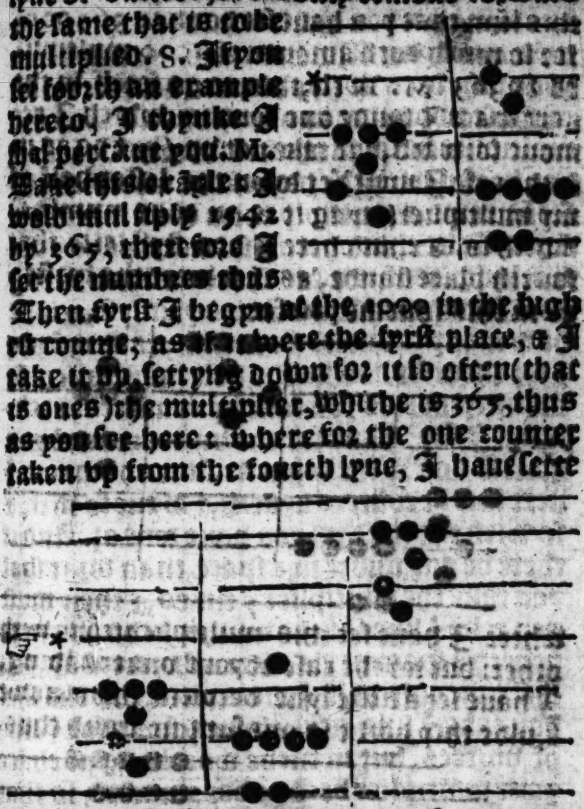


beste,

beſt,ſo ſom like the other: therfore you now knowyng both,may vſe which you lyſt. But now touchyng Multiplicatiō: you ſhall ſet your numbres in two romes (as you dydde in thoſe two other kyndes) but ſo that the multiplier be ſet in the firſte rome. Thē ſhall you begyn with the highest numbre of the ſet ſo rome,& multiply them fyrſt after this ſorte. Take that outermoſt line wt your firſte worſkyng,as if it were the lowreſt line,ſetting on it ſom moueable marke as you lyſt,& loke how many counters be in him,take thē vp, & for them ſet downe the holle multiplier,ſo manytymes as you toke vp counters,reckening(I ſay) ỳ line for the vnites:& whē you haue ſo don wt the highest nūbre,thē com to ỳ next line of coūters beneth,& wt cū ſo ỳ is,& ſo with the next ỳ you haue done.&. And if there be any nūbre in a ſpace,than for it ſhal you take the multiplier 5 times,& then muſt you reke ỳ line for the vnities,which is next beneth ỳ ſpace:or els after a ſhorter way,you ſhal take only halfe ỳ multiplier,but thā ſhal you take ỳ line next aboue ỳ ſpace for ỳ line of vnitees,but in ſuche worſkyng,if thāce your multiplier be an odde numbre,ſo that you can not take the halfe of it iuſtly,than muſte you take the leſſer halfe,and ſet down that,as yf that it were the iuſte halfe,and farther you ſhal ſet one counter in the ſpace beneth that lyne,whiche you reckon for the
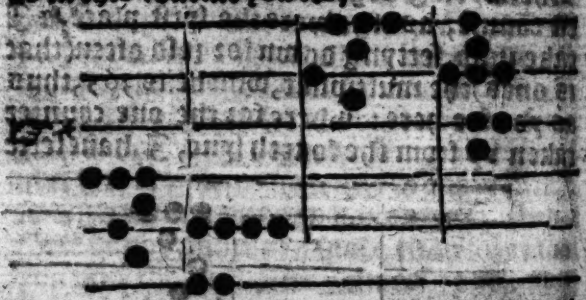
Multiplica-
tion.

lyne of vnitees, as is only remoue forwarde the same that is to be multiplied. 8. Ifyou set forth an example hereto, I shynke I shall perfaue you. M. Make thys lesse sure: I wold multiply 1542 by 365, therfore I set the numbers thus



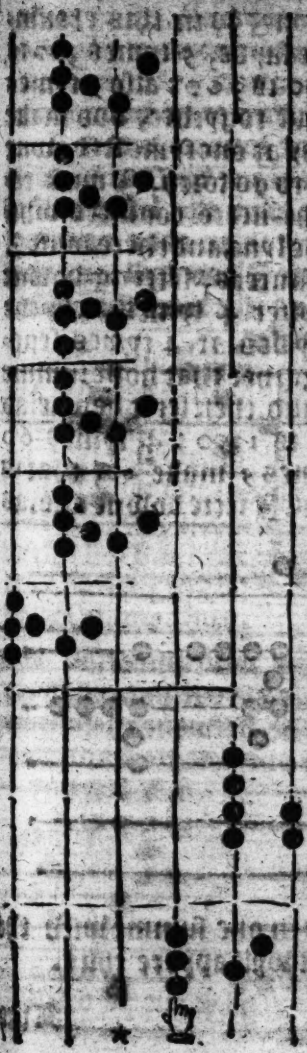
Then fyrst I begyn at the 2 in the highest rounde, as if 2 were the fyrst place, & I take it vp, settyng down for it so often (that is ones) the multiplier, whiche is 365, thus as you see here: where for the one counter taken vp from the fourth lyne, I haue sette downe other 6, whych make the sum of the multiplier, reckenynge that fourth lyne as yf it were the fyrst, whiche thyng I haue marked by the hand set at the begynnyng of the same,

Lowe. 8. I perceaue this well, for in deede, this ſumme that you haue ſet downe is 365000, for ſo much doth amount of 1000 multiplied by 365. M. Well, then to go forth, in the next place I finde one vnmeete, which I muſt remoue forward, but take nat vp, but do as in ſuche caſe I muſt. Set down the leſſer half of my multiplier (ſeyng it is an odde number) whych is 182, and here I doo ſtyll letter that fourth place ſtande, as if it were the firſte, as in this fourme you ſee,



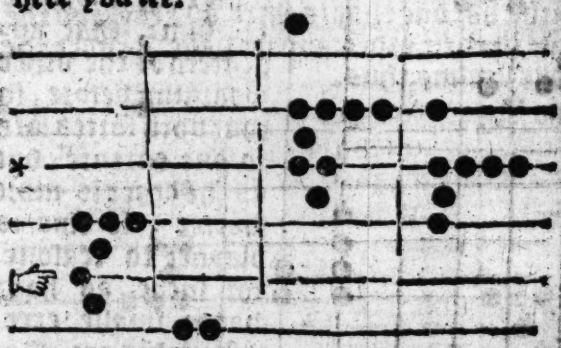
where I haue ſet this multiplication with other: but for the eaſe of your vnderſtāding, I haue ſet a little lyne betweene them: nowe ſhulde they both be in one ſumme ſtand thus.





Howe be it, an other fourme to multiplye suche nombers in space is this: Fyrste to remoue the finger to the fyne nexte beneth that space, and thanne to take vppe that nombre, and to sette downe the multiplyer fyue tymes, as heere you see. Whyche summes if you doo adde togyther into one summe, you shal perceaue that it wyll be the same, that appeareth of the other wourking before, so that both sortes are to one entente, but as the other is more shorter, so this is playner to reasone, for suche as haue hadde smalle exercise in this art. Notwithstandynge you maye adde theim in your mynde beefore

you sette theym downe, as in this exaumple you myghte haue sayde, 5 tymes 300 is 1500, and 5 tymes 60 is 300: also 5 tymes 5 is 25, whiche all put togyther, doo make 1825, whiche you may at one tyme set downe if you lyst. But now to go forth, I must remoue the hande to the nexte counters whiche are in the seconde lyne, and there must I take by those 4 counters, settyng downe for theim my multiplier 4 tymes, whyche thynge other I may doo at 4 tymes seuerally, or els I may gather that holle summe in my myne eyght, and then set it downe: as to say, 4 tymes 300 is 1200: 4 tymes 60 are 240: and 4 tymes 5 make 20, that is in all 1460, that shall I sette downe also, as here you see.

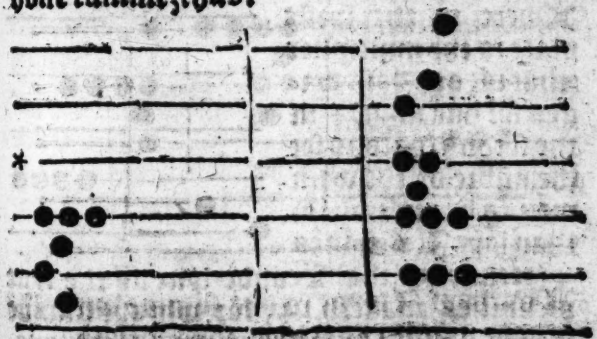


Whyche yf I toyne in one summe with the former numbres, it wyll appere thus.

Then



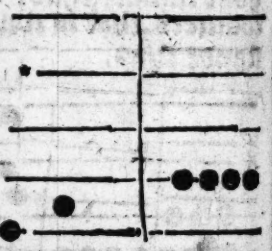
Then to end this multiplication, I remoue the tynger to the lowest lyne, where are only 2, them doo I take vp, and in their stede do I set downe twyse 365, that is 730, for which I set one in the space aboue the third lyne for 500, and 2 more in the thyrde lyne with that one that is there all redy, and the reste in their order, and so haue I ended the holle summe, thus.



Wherby you see, that 1542 (whyche is the

ies sith ristes in nation.

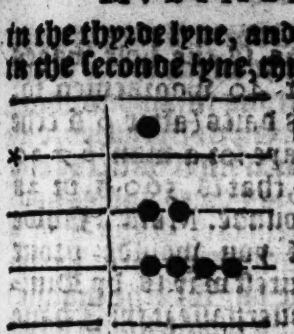
numbre of yeres sithe Christes incarnatiō) being multiplied by 365 (which is the numbre of daies in one yere) doth amount vnto 562830, whyche declareth the numbre of daies sith Christs incarnation vnto the end of 1542 yeres (beside 385 daies and 12 houres for leape yeres.) Scholar. Nowe wyll I proue by an other exaumple, as this: 40 labourers (after 6 d the day for eche manne) haue wrought 28 daies, I would know what their wages dothe amounte vnto. In this case muste I woorke doubliely: fyrst I must multiply the numbre of the laborers by the wages of a manne for one daie, so will the charge of one daie amount: then secondarily shall I multiply that charge of one daye by the whole numbre of daies, and so wil the holle summe apere: fyrst therfore I shal set the summes thus.



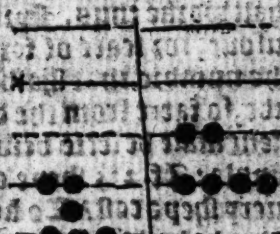
Where in the fyrste place is the multiplier (that is, one daies wages for one man) & in the second space is set the nūbre of the workmen to be multiplied: than saye I, 6 tymes 4 (reckenynge that seconde lyne as the lyne of vnities) maketh 24, for whyche summe I should sette 2 counters in the thyrde lyne, and 4 in the seconde, therfore doo I sette 2

in

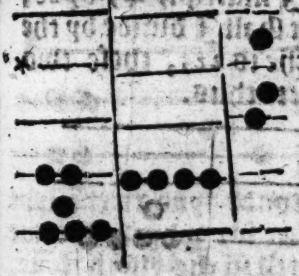
in the thyrde lyne, and lette the 4 stande still in the seconde lyne, thus.



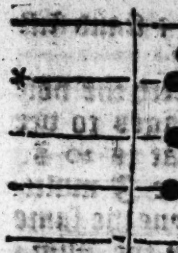
So appereth the hole dayes wages to bee 240 d, that is 20 s. Then doe I multiplie agayne the same summe by the numbre of dates, and firste I lette the numbres,



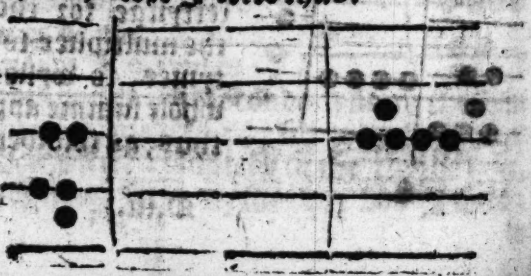
thus, as foloweth. Then bycause there ar counters in diuers lynes, I shall begyn with the highest and take theym vp, settynge for theym the multiplyer so manny tymes as I take vp counters, that is twise, then vpll the summe stande thus,



Then come I to the seconde lyne, & take vp those 4 counters, settynge for theym the multiplier fowre tymes, so vpllie the whole summe appere thus, as foloweth.



So is the hole wages of 40 woorkemen for 28 daies (after 6 d eche daye for a man) 67 10 d, that is 560 s, or 28 pounde. Maist. Nowe if you woulde proue Multiplication, the surest way is by Diuision: therfore will I ouerpasse it tyl I haue taught you the arte of Diuision, whiche you shall woorke thus. Fyrst, lette downe the Diuisour, for feare of forgettyng, and than set the numbre that shalbe diuided, at the right side, so farre from the diuisour, that the quotient male be serte betweene them: as for example: If 225 sheepe cost 45 li, what dyde euery sheepe cost? To knowe this, I should deuide the holle summe, that is 45 li by 225, but that can not be, therfore must I fyrst reduce that 45 li. into a lesser denomination, as into shillyngs: then I multiply 45 by 20, & it is 900, that summe shall I diuide by the numbre of sheepe, whiche is 225, these two numbres therfore I serte thus.



Then begyn I at the byghest lyne of the dy-uident, and seke how often I may haue the diuisour therein, and that may I doo 4 tymes, then say I: 4 tymes 2 are 8, whych yͤ I take from 9, there resteth but 1, thus.



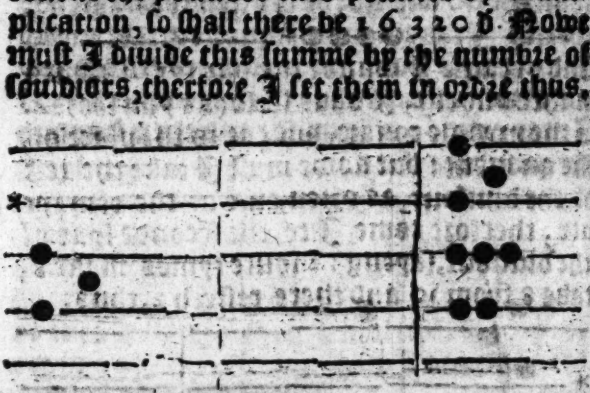
And bycause I founde the diuisor 4 tymes in the diuident, I haue sette (as you see) 4 in the myddle rowme, whiche is the place of the quotient: but nowe must I take the rest of the diuisour, as often out of the remay-ner, therfore come I to the seconde lyne of the diuisour, sayeng: 2 foure tymes make 8, take 8 from 10 and there resteth 2, thus.



Then come I to the lowest numbre, whiche is 5, and multiplye it 4 tymes, so is it 20,
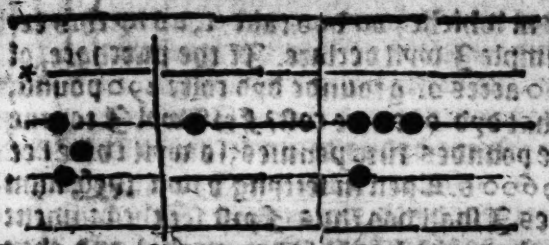
that take I from 20, and there remaynyth nothyng, so that I see my quotient to be 4, whiche are in valewe shyllynges, for so was the diuident: and therby I knowe, that yf 225 sheepe dyd coste 45 li, euery sheepe coste 4 s. Scholar. This can I doo, as you shall perceaue by this exaumple: If 160 souldiours doo spende euery moneth 68 li, what spendeth eche man? First bycause I can not dyuide the 68 by 160, therfore I wyll tourne the poundes into pennies by multiplication, so shall there be 16320 d. Nowe must I diuide this summe by the numbre of souldiors, therfore I set them in ordre thus.

mple of 4 s. iours 8.
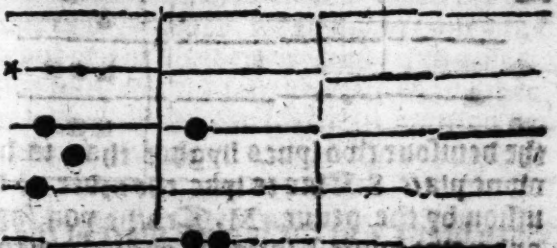


Then begin I at the highest place of the diuident, seeyng my diuisour there, whiche I fynde ones, therefore set I 1 in the nether lyne. Maist. Not in the nether lyne of the holle summe, but in the nether lyne of that woorke, whiche is the thyrd lyne. Scho. So standeth it with reason. M. Then thus doo they stande.

Then

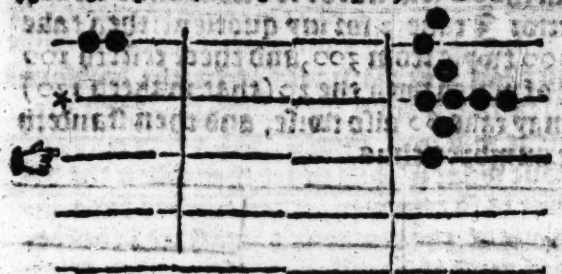


Then ſeeke I agayne the reſte, howe often I maye fynde my diuiſor, and I ſee that in the 300 I myght fynde 100 three tymes, but than the 60 will not be ſo often found in it, therfor I take 2 for my quotient: then take I 100 twyſe from 300, and there reſteth 100 out of which with the 20 (that maketh 120) I may take 60 alſo twiſe, and then ſtandeth the numbres thus.



where I haue ſette the quotient 2, in the loweſt lyne: So is euery ſolidiours portion 102 ℔, that is 8 ℥, 6 ℔. Maiſter. But yet becauſe you ſhall iuſtely perceaue the reaſon of Diuiſion, it ſhall be good that you doo ſet your diuiſour ſtyll agaynſte thoſe numbres
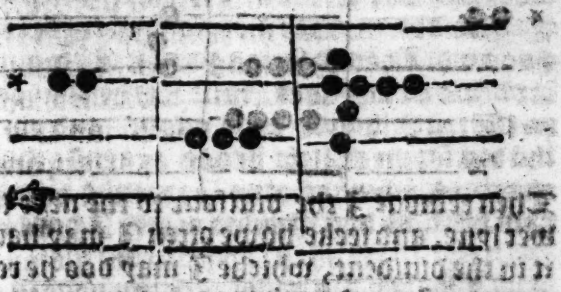
ample pour asse. from whiche you doo take it, as by this example I wyll declare. If the purchace of 200 acres of grounde dyd coste 290 pound, what dyd one acre coste? Fyrst wyl I reduce the poundes into pennies, so wyll there bee 69600 d. Then in settyng down these numbres I shall doo thus. Fyrst set the diuident on the ryght hande (as it ought) and then the diuisor on the left hande agaynste those numbres, frome whyche I intende to take hym fyrst, as here you see, where I haue set



the deuisour two lynes hygher than is his owne place. S. This is lyke the order of diuision by the penne. M. Truthe you saye, and nowe muste I sette the quotient of this worke in the thirde lyne, for that is the lyne of vnites in respecte to the diuisour, in this worke. Then I seke howe often the diuisor may be founde in the diuident, and that I fynde 3 tymes, then sette I 3 in the thyrde lyne for the quotiente, and take awaye that

60000

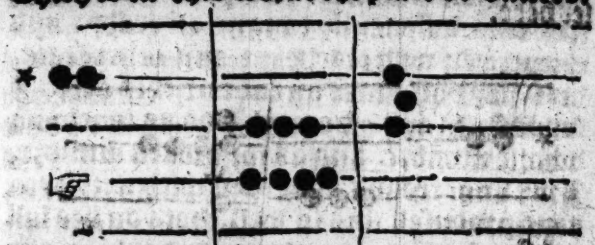
60000 frome the dyuidente, and farther I doo sette the diuisour one lyne lower, as you see here.



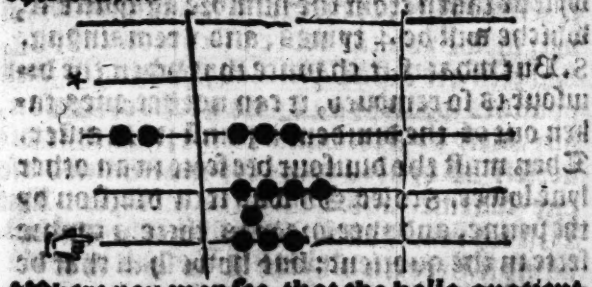
And then sethe I howe often the diuisour wyll be taken from the numbre agaynste it, whiche will be 4 tymes, and 1 remaynyng. But what if it chaunce that when the diuisour is so remoued, it can not bee ones taken out of the dyuidente agaynst it? Master. Then must the diuisour bee sette in an other lyne lower. Scoler. So was it in diuision by the penne, and therfore was there a cypher sette in the quotient: but howe shall that be noted here? M. Lytle nedeth no token, for the lynes do represente the places: only take that you sette your quotiente in that place, whiche standeth for vnytes in respect of the diuisour: but nowe to returne to the exãple, I find the diuisor 4 tymes in the dyuidēt, & there remaynyng, for 4 tymes 2 makes 8, whiche I take frō 9, & there resteth 1, as this figure to lowynge sheweth: & in the myddle space for

the

the quotient I sette 4 in the seconde lyne, whiche is in this worke the place of vnites.



Then remoue I the diuisour to the nexte lower lyne, and seeke howe often I may haue it in the diuident, whiche I may doo here 8 tymes iuste, and nothynge remayne, as in this forme

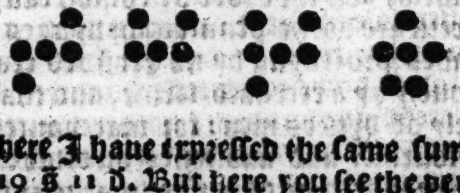


Where you may see, that the holle quotient is 348 ℔, that is 29 ℥, whereby I knowe that so muche coste the purchase of one acre. Now resteth the proues of Multiplication, and also of Diuision. M. Their best proues are eche one by the other, for Multiplication is proued by Diuision, and Diuision by Multiplication, as in the worke by the

the peine, you learned. Scolar. If that be all, you shall not nede to repete agayn that, that was sufficiently taught all ready: and except you wyll teache me any other feate, here may you make an ende of this arte, I suppose. Mayst. So wyll I do as touchyng whole numbre, and as for broken numbre, I will not trouble youre wyt with it, tyll you haue practised this so well, that you bee full perfect, so that you nede not to doubt in any point that I haue taught you, and thā may I boldly instruct you in the art of fractions or broken numbre, wherin I wyll also shew you the reasons of all that you haue nowe learned. But yet before I make an ende, I will shewe you the order of cōmon castyng, wherein are bothe pennies, shyllynges, and poundes, procedyng by no grounded reason, but onely by a receaued forme, and that diuersely of dyuers men: for marchantes vse one fourme, and auditours an other: But fyrst for marchantes forme, marke this example here, in whyche I ● ●●●● haue expressed this summe ● 198 li, 19 s, 11 d. So that ● ●●● you may se, that the lowest ● line serueth for pēnies, the ● ●●●● nexte aboue for shillynges, ● the third for poundes, & the ● ●●●● fourth for scores of poūdes. And farther you may se, that the space betwene pens & shillyngs

may

maye receaue but one counter (as all other spaces lykewayes dooe) and that one standeth in that place for 6 d, Lykewaies beeyng the shillynges and the pounds, one counter standeth for 10 s. And betwene the poũdes and 20 li, one counter standeth for 10. poundes. But beside those you may see at the left side of shillynges, that one counter standeth alone, and betokeneth 5 s. So against ½ poũdes, ½ one counter standeth for 5 li. And against ½ 20 poundes, the one coũter standeth for 5 score poundes. ½ is 100 li, so ½ euery side coũter is 5 tymes so much as one of thē against which he standeth. Now for ½ accompt of auditors take this exãple.



Where I haue expressed the same sum 198 li, 19 s 11 d. But here you see the pennies stande towarde the right hande, and the other encreasynge orderly towarde the lefte hande. Agayne you may see that auditours will make 2 lynes (yea and more) for pennies, shillynges, and all other valewes, yt theyr summes extende therto. Also you see, that they set one counter at the right end of ethe rowe, which so set, there standeth for 5 of that roume: and on the left corner of the

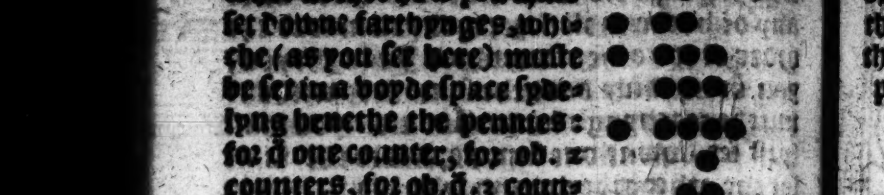
rowe ys standeth for 10 of the same rowe. But now if you wold adde other subtracte after any of bothe these sortes, if you marke the order of the other feat which I taught you, you maie easily doo the same here without muche teachyng: for in Addition you must fyrst set downe one summe, and to the same set the other orderly, and lyke maner if you haue many: but in Subtraction you muste sette downe fyrste the greatest summe, and from it must you abate that other, euery denomination from his owne place. S. I doo not doubt but with a lyttell practise I shall attaine these bothe: but howe shall I multiply and diuide after these formes? M. You can not duely doo none of bothe by these sortes, therefore in suche case, you must resorte to your other artes. S. Syr, yet I see not by these sortes howe to expresse hundredes, yf they excede one hundred, nother yet thousandes. M. They that vse suche accomptes that it exceede 200 in one summe, they sette no 5 at the left hande of the scores of pounes, but thei set al the hundreds in an other farther rowe, and 500 at the left hand therof, and the thousandes they set in a farther rowe yet, and at the left syde therof they set the 5000, and in the space ouer they set the 10000, and in a hygher rowe 20000, why the all I haue expressed in this exaumple, whiche is 9 7 8 6 9 li 12 s, 9 d ob. q. for

I had

I had not told you before, whether nother how you shuld set downe farthynges, whiche (as you see here) muste be set in a voyde space sydelyng benethe the pennies: for ♁ one counter, for ob. 2 counters, for ob ♁ 3 counters, and more there can not be, for 4 farthyngs do make 1 ♁, whyche muste be in his newe place.



And if you desire the same summe after auditours maner, lo here it is.



But in this chyng, you shal take this for sufficient, and the reste you shall obserue as you maie see by the worckyng of eche sorte: for the diuers wittes of men haue inuented diuers and sundry waies almost vnnumerable.

But one feate I shalll teache you, whyche not only for the strangenesse and secretenes is muche pleasant, but also for the good commoditie of it, tyghte worthy to be well marked. This feate hath been vsed aboue 2000 yeres at the least, and yet was it neuer commonlye knowen, especially in Englyshe it

was

was neuer taught yet. This is the art of nūbryng on the hand with diuers gestures of the fingers expressyng any sum conceiued in the mynd. And fyrst to begyn: If you wil expresse any sum vnder 100, you shal expresse it with youre lefte hande: and fro me 100 vnto 10000, you shal expresse it with youre ryght hande, as here orderly by this Table folowyng you may perceaue.

Here folowerh the table of the arte of the hande.


10
100
1000
2
20
200
2000
3
30
300
3000
4
40
400
4000
5
50
500
5000
6
60
600
6000
7
70
700
7000
8
80
800
8000
9
90
900
9000

1 In whyche (as you may see) 1 is expressed by the lyttle fynger of the left hand closely and hard crooked. 1

2 is declared by lyke bowynge of the weddyng fynger (whych is the nexte to the lyttle fynger) together with the lyttle fynger. 2

3 is signified by the myddle fynger bowed in lyke maniere with those other two. 3

4 is declared by the bowynge of the myddle fynger, and the ryng fynger or weddyng fynger, with the other all stretched forth. 4

5 is represented by the myddle fynger onely bowed. 5

And 6 by the weddyng fynger onely crooked: and thys you may marke in these certayne order. But nowe 7, 8, and 9, are expressed wyth the bowyng of the same fyngers, as are 1, 2, and 3, but after an other fourme. 6

For 7 is declared by the bowyng of the lyttell fynger, as is 1, saue that for 1 the fynger is clasped in, harde and rounde, but for to expresse 7, you shall bowne the myddle ioynt of the lyttle fynger onely, and holde the other ioyntes streyght. Scho. If you wyll geue me leaue to expresse it after my rude maner, thus I vnderstand your meanyng, that 1 is expressed by crookynge in the lyttle fynger, like the heed of a bishops bagle: and 7 is declared by the same fynger bowed, like a gybbet. Ma. So I perceaue, you hu-

derstand it.

8 When to expresse 8, you shall bowe after the same maner both the lyttle fyngar and the ryng fynger.

9 And if you bowe lykewyse with thyne the myddle fynger, than doth it betoken 9.

10 Now to expresse 10, you shall bowe youre forefynger roundе, and set the ende of it on the hyghest toynt of the thombe.

20 And for to expresse 20, you must sette your fyngers straight, & the ende of your thombe to the partition of the foremoste and myddell fynger.

30 30 is represented by the toyning togither of the headdes of the foremoste fynger and the thombe.

40 40 is declared by settynge of the thombe crosselwayes on the formoste fynger.

50 50 is signyfied by eyghe stretchyng forthe of the fyngers ioynctly, and applyyng of the thumbes ende to the partition of the myddle fynger, and the ryng fynger or weddyng fynger.

60 60 is fourmed by bendyng of the thombe crooked, and crossyng it with the forefynger.

70 70 is expressed by the bowyng of the formoste fynger, and settynge the ende of the thombe betweene the 2 formoste or highest toynts of it.

80 80 is expressed by settynge of the fore-

moste fynger crosselwayes on the thombe, so that 80 differeth thus from 40, that for 80 the forefynger is sette crosselwayes on the thombe, and for 40 the thombe is set crosse ouer the forefynger.

90 is signified by bendynge the forefynger, and settyng the ende of it in the innermoste ioynte of the thombe, that is euen at the sote of it. And thys ar all the numbres ended vnder 100. Indede these be all the numbres from 1 to 10, and then all the tenthes within 100, but thys teached me not how to expresse 11, 12, 13 &c. 21, 22, 23, &c. and suche lyke. M. You can lytell vnderstande, yf you can not doo that without teachynge. What is 11? is it not 10 and 1? then expresse 10 as you were taught, and also and that is 11: and for 12 expresse 10 and 21 for 23 sette 20 and 3: and so for 68 you muste make 60 and there to 8: and so of all other sortes.

But now if you would represente 100, & ther any numbre aboue it you must doo that with the ryght hande, after this maner.

You must expresse 100 in the ryght hand with the lytle finger so bowed, as you did expresse 1, in the lefte hande.

And as you expressed 2 in the left hand, the same fashion in the ryght hande dooth declare 200.

The fourme of 3 in the ryght hand standeth for 300.

90

11, 12

21, 22

100

200

300

400 The fourme of 4, for 400.

500 Lykewayes the fourme of 5, for 500.

600 The fourme of 6, for 600. And to declare to you how you shyll expresse single vnites and tenthes in the lefte hande, so muste you expresse vnites and tenthes of hundredes, in the ryght hand. S. I vnderstand you thus: that if I woulde represent 900, I muste so fourme the fyngers of my ryght hand to expresse 9; And as in my lefte hand I expressed 10, so in my ryght hand must I expresse

900

1000 1000.

4000 And so the fourme of euerye tenth in the lefte hande, serueth to expresse like numbre of thousandes, so the fourme of 40 standeth for 4000.

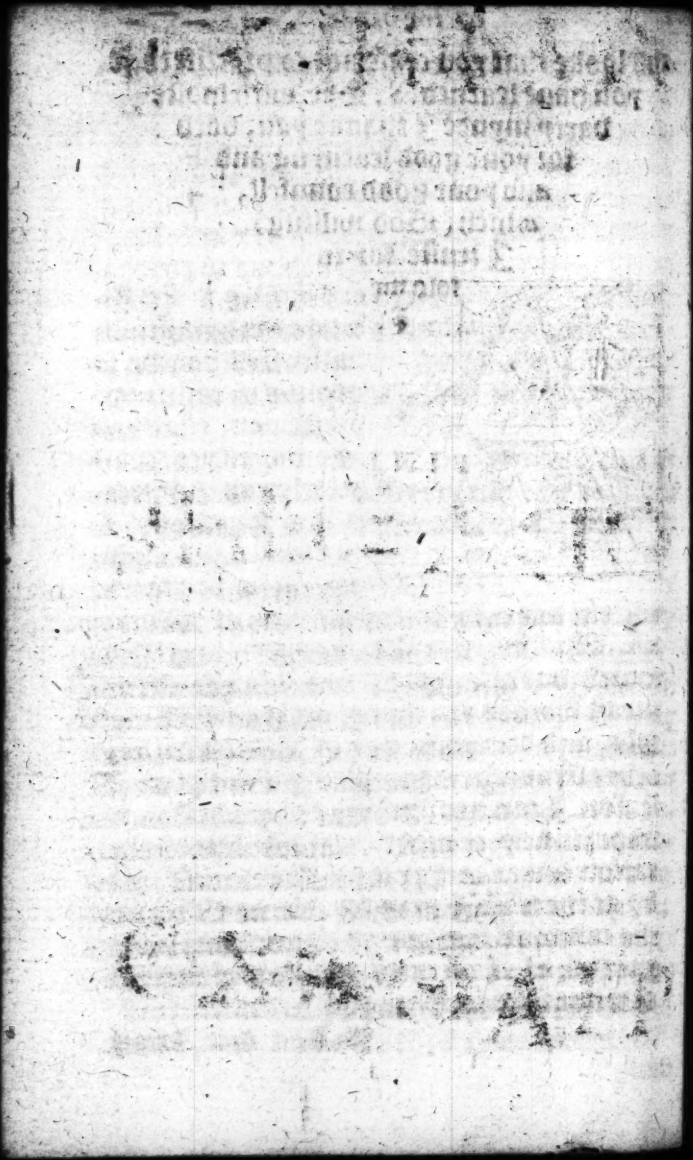
8000 The fourme of 80 for 8000.

9000 And the fourme of 90 (whych is the greatest) for 9000, and aboue that I can not expresse any numbre. M. No, not with one fingers how be it, wyth diuers fingers you may expresse 9999, and all at one time, and that lacketh but 1 of 10000. Forthat vnder tenne thousande you maye by your fyngers expresse anye summe. And this shall suffice for Numeration on the fyngers. And as for Addition, Subtraction, Multiplication, and Diuision (whyche yet were neuer taught by any man as farre as I do know) I wyll instructe you after the treatise of fractions, and nowe for thys tyme fare well.

and

and looke that you cease not to practise that you haue learned. S. Hye, with moste harty mynde I thanke you, both for your good learnyng and also your good counsell, which (God willing) I truste for to folowe.

FINIS.



THE SECOND PART OF ARITHMETIKE TOVchyng Fractions, bretely set forthe.

Scholar.



ALBEIT I PERceyue your manifolde businesses boothe to occupie or rather oppresse you, that you can not as yet completely end that treatise of Fractions Arithmeticall, whiche you haue prepared, wherin not only sundry workes of Geometry, Musike, and Astronomie be largely set forth, but also diuers conclusions and naturall workes touchyng mixtures of metals, and compositions of medicines, with other straunge examples, yet in the meane season I can not staie my earnest desyre, but importunely crane of you some briefe preparation towarde the vse of Fractions, whereby at the least I may be able to vnderstand the common workes of them, and the vulgare vse of those rules, which without them can not well be wrought.

Mai. Maist.

Maister. If my leysure were as great as my wyll is good, you shuld not nede to vse any importunate prayyng for the attaynynge of that thynge, whereby I may bee perswaded that I shall any wayes profite the common welthe, or healpe the honest studies of any good membres in the same, wherfore whyle myne attendance wyll permyt me to walke and talke, I am well wyllynge to helpe you as I may.

What after this. Therfore fyrste to begyn with explication of this name Fraction, What take you it to bee?

Scholar. Mary syr I thynke a fraction (as I haue herde it often named) to be a broken numbre, that is to say, to be no hole numbre but a parte of a numbre.

Maist. A fraction in dede is a broken numbre, and so consequently the part of an other numbre, but that must be vnderstand of such an other noumbre, as can not be diuided into any other partes then fractions, for al though I may take the thyrd part of 60, or the fourth parte of it, and so of other partes diuersely, yet these partes bee not properly, nor ought not to be called fractions, bicause they may be expressed by hole numbres, for the thyrde part of it is 20, the fourth parte is 15, the twelfth parte is 5, and so forth of other partes, whiche all be hole numbres.

Wherefore properly a fraction expresseth the

the partes or part only of an vnitie, that is to say, that the numbre whiche is the hole or intiere summe of any fraction, may not be greater then one, and therfore it followeth, that no one fraction alone can be so great, that it shall make 1, as by examples I will declare, as soone as I haue taught you to knowe the fourme how a fraction is expressed or representeth in writyng.

NVMERATION.

But fyrst to begyn with the expressyng of a fraction, whiche is the Numeration of it, you must vnderstand, that a fraction is representeth by 2 numbres, set one ouer the other, and a lyne drawen betweene them, as thus: $\frac{1}{2}$ $\frac{3}{4}$ $\frac{2}{5}$ $\frac{1}{17}$. whiche foure fractions you muste pronounce thus: $\frac{1}{2}$, one thyrde parte: $\frac{3}{4}$ three quarters: $\frac{2}{5}$, two fyft partes: $\frac{1}{17}$, &. seuentene partes. Numeration of fractions.

Scho. I vnderstand the forme of their expression and pronunciation, but their meanyng or valuation semeth more obscure, yet I thynke, that by the. ij. fyrste fractions I vnderstande the valuation of the. ij. later fractions, and so consequently of other.

Mast. Maiew them than, that I may perceaue your fantyngs of them.

Scho. $\frac{2}{5}$ betokeneth two fyft partes, that is to saie, if one bee diuided into 5 partes, that fraction doth expresse ij. of them, so $\frac{1}{17}$ dothe

doth signifie, that if one be diuided into .iiij. partes, I must take .i. of them. And this I gather of the .ii. fyrst examples. for ¼, that ys one thyrd parte, doth easyly declare, that if any one thyng be deuyded into .iij. partes, I must take but one of them. so ¾, that is .iij. quarters, doth declare that one beyng deuyded into .iij. quarters, I must take for this fraction .iij. of thole quarters.

And if there bee nomore difficultye in the Numeration, then I pray you go forward to their addition and subtraction, and so to the other kinds of workes, for I vnderstand that the same kyndes of workes be in fractions, that bee in hole numbres.

Mai. There are the same kynde of workes in bothe, albeit the order of them is diuers, as I will anon declare, but yet more in Numeration (before we leaue it) you muste vnderstande, that those .ij. numbres, whiche expresse a fraction haue seuerall names, the ouermoste, which is aboue the lyne, is called the Numerator, and thother beneth the lyne is called the Denominator.

Numerator
Denominator

Scho. And what is the reason of theyr diuers names? for in my opinion bothe bee numeratours, seyng both they doo expresse the numeration of the fraction.

M. You are deceyued, for one only (which is the ouermoste) bothe expresse the numeration, and the denominator doth declare the

numbre

numbre of partes, into whiche the vnitie is diuided, as in this example, whē I saye: Diuide a pounde weighe of golde betwene.iiij. men, so that the fyrst man shall haue $\frac{2}{15}$, the second $\frac{3}{15}$, the thyrd $\frac{4}{15}$, and the fourth $\frac{6}{15}$. Nowe doo you perceaue that by the denominator (whiche is one in all. iiii. fractions) it is intended, that the pound weyght shuld bee diuided into so many partes, I meane 15, and by the iiii. seuerall numeratours is limitted the dyuers portion that eche man shal haue, that is, $\frac{2}{15}$ when the hole is parted into 15, the fyrst man shall haue 2. of those 15 partes, the seconde man 3 of theym, the thyrde man 4, and the fourthe man 6. And so may you se the seuerall offices (as it were) of those ii. numbres, I meane of the numerator and the denominator.

And hereby you perceaue, that a man can haue no more partes of anye thynge, then it was dyuided into, nother yet aptly so many, so that it were vnaptly sayd, you shall haue $\frac{15}{15}$ that is xb. fyftene partes of any thynge, seyng it were better sayd. You shal haue the hole thyng. S. So doth it appeare reasonably, for the labour is vayne to dyuide anye thyng, and then to apply the diuision to no vse. And much lesse reasonable were it to say $\frac{15}{15}$. for yf the hole be dyuided into 15 partes only, it is not possible to take 16 of them, that is to say, more then altogether. M. This is true, touchyng the proper and apte vse of the name

the name of a fraction, yet improperly, and after a vulgare acceptation (for easynes in woorke) bothe those formes be called fractions, bicause they be written lyke fractions, although they be none in deede, for $\frac{1}{1}\frac{5}{2}$, and generally all suche other where the numerator and denominator be equall, are not fractions, but the hole thynge with all his partes. And so $\frac{1}{1}\frac{5}{2}$ is not to be called a fraction, but a myxte numbre of a hole numbre and a fraction, for it is as muche as $1\frac{5}{12}$ that is one hole one, and 4 twelfe partes, as shall be declared in Reduction. Therfore they do abuse the names that call them fractions, where the numerator is other equall or greater than the denominator. Scho. But is there any necessarie cause why they should so abuse the name? Maister. There is cause why they shall sometymes for easynesse in woorke, write somme noumbres after that sorte lyke fractions, but they needed not to calle theym fractions, but as they bee, hole numbres, or myxte numbres (that is hole numbres with fractions) expressed lyke fractions.

Now muste you understande, that as no fraction properly can bee greater then 1, so in smalnesse under one, the nature of fractions dooth extend infinitely, as the nature of hole numbres is to increase above one infinitely, so that not onely one maye be diui-

ded

ded into infinite fractions or parts, but also euery fraction maye bee diuided into infynite fractions or partes, whiche commonly bee called fractions of fractions, and they bee expressed dyuersely: As for exaumple: $\frac{3}{4}\ \frac{2}{3}\ \frac{1}{2}$ that is, iii. quarters of two thyrds partes of one halfe parte, whereby is signified, that if one be diuided into two haines, and the one halfe into three partes, and. ii. of those iii. partes be diuided equerely into iiii. quarters, this fraction dooth represent iii. of those quarters. Scholar. I pray you lette me proue by an exaumple in common monney, whether I dooe ryghtely vnderstande you or no. One crowne (whyche I take for an vnitie) dooeth conteyne 60 pennies, therefore the halfe of it is 30 pence, $\frac{2}{3}$, of that halfe is 20 pence, whereof $\frac{3}{4}$ is 15 pence, so than 15 pence is $\frac{3}{4}\ \frac{2}{3}\ \frac{1}{2}$ of a crowne. And so 3 pence is $\frac{3}{4}\ \frac{2}{3}\ \frac{1}{2}$ of a shillynge. Maister. You perceaue this well inough, but how hapned that you found no doubte in the forme of writyng theise fractions? seyng the two later fractions haue no line betwene theyr numbres as the fyrst hath? S. Bycause I had forgoten (as scholars oft tymes do) that, that was told me before, but I pray you expresse the reason thereof Mai. This forme is but voluntary, & therfore hath none other reson then I will of I dunstor, whyche forme many doo folowe. Some other do

make

make lynes betwene euery fraction, and adde wordes of diuision after this forte, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$, whiche forme is good also, some other expresse theim thus, $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ in slope forme, to distinct theim from seuerall fractions of one hole numbre. for yf they were sette in one right line thus, $\frac{3}{4}$ $\frac{2}{3}$ $\frac{1}{2}$ than ought it to be pronounced. iii. quarters, and .ii. thyrde partes, and an half, whiche maketh almoste .ii. hole vnities, lacking but one .xii. part. And so is it nothyng agreable with the other fraction of fractiõs wherfore it is a great ouersyght in certayne learned men, whiche do expresse them so confusedly with suche seuerall fractions, that a man can not knowe the one from the other. Scho. Than I perceaue there be .iii. seuerall varieties in fractions, yet whan one onely fraction is sette for one numbre, as $\frac{4}{5}$, that is. iiii. fyfte partes. the second is, when there be sette .ii. or more seuerall fractions of one numbre, as $\frac{4}{5}$ $\frac{3}{7}$, that is, iiii. fyfte partes of .ii. vii parts. the thirde sorte of fractions of fractions, as $\frac{4}{5}$ $\frac{2}{3}$, that is iiii. fyfte partes of .ii. thyrde partes. M. You haue sayd well, if you vnderstande well your owne wordes. S. If it shall please you I wyll by an example in the partes of an olde englyshe angell expresse my meanynge. Mast. Do so I praye you. Scho. The olde englyshe angell dyd contayne vii. s. vi. d. that is 90 d. Now $\frac{2}{3}$ of it

is 72 d. And of the same 90 d, yf I take $\frac{4}{5}$ $\frac{7}{9}$ that is iiii. nynthe partes, and ii. fyfte partes, $\frac{4}{5}$ is 40, and $\frac{7}{9}$ is 36, whiche both make 76, but yf I take $\frac{4}{5}$ $\frac{7}{9}$, that is iiij. nynthe partes of.ij. fyfte partes, seyng $\frac{7}{9}$ is but 36, then $\frac{4}{5}$ of 36 wyll yelde but 16, for $\frac{1}{5}$ of 36 is but 4, and that taken iiij. tymes, maketh 16. Maister. This is plainely expressed and truely, and hereby (I doubte not) but you doo perceaue, that as greate a difference as is betwene 16 and 76, so muche difference is betwene these ij. fractions $\frac{4}{5}$ $\frac{7}{9}$ and $\frac{4}{5}$ $\frac{7}{9}$. And nowe that you vnderstande these varietes, I wyll procede to the reste of the wozkes, fyrst admonyshyng you that there is an other ozder to be folowed in fraction, then there was in hole noumbzes, foz in hole numbzes this was the ozder, Numeration, Addition, Subtraction, Multiplication, Diuisyon, and Reduction, but in fractions (to folowe the same aptenesse in procedynge from the ealyest wozkes to the harder) we muste vse this ozder of the wozkes, Numeration, Multiplication, Diuision, Reduction, Addition, and Subtraction. Scholar. That Multiplication and Diuisyon shoulde go togyther, and Subtraction to folowe Addition, naturall ozder dothe perswade, but why Multiplication shuld be fyrste in ozder here nexte to Numeration, and Reduction in the myddle, I desyre to

The ozder of wozkes in fractions

vnderstande the reason. Master. As in the grose of hole numbres order would reasonably begyn with the easiest, and so go forward by degrees to the hardeste, euen so reason teacheth in Fractions the lyke order. And consyderyng that Addition or Subtraction of fractions can very seldome bee wrought without Multiplication and reduction: and contrarywaies Multiplication and reduction may be wrought without this form of Addition or Subtraction, therfore was it orderlye required, that Multiplication and reduction shoulde go before Addition and Subtraction. And the same reason serueth for the placynge of Multiplycation before Reduction. Scholar. Then yf Multiplycation be the easyest, I praye you declare the fourme of it, fyrst by rule, and then by example. Mai. Your request is good.

MVLTIPLICATION.

Therefore when any two fractions bee proponed, to bee multiplied togyther, the numerator of the one must be multiplied by the numerator of the other, and the sum that amounteth therof, must be set for a new numeratour; lykewaies the denominatour of the one must bee multiplied by the denominator of the other, and that that amounteth shall bee sette for the common denominatour, and this newe fyrde fraction re-

preſteth the multiplication of the two fyrſte fractions proponed, wherof take this exam ple, $\frac{3}{5}$ multiplied by $\frac{5}{12}$, doth make $\frac{15}{60}$. S. I perceaue then that 3 beynge the numera tour of the fyrſt fraction, is multiplied by 5, beynge the numeratour of the ſecond frac tion, wherof amounteth 15, the numerator of the thyrde fraction And ſo lykewaies 5 beynge denominatour of the fyrſt fraction, is multiplied by 12, the denominatoure of the ſeconde fraction, wherof amounteth 60, the newe denominatour, ſo that I perceaue howe the wooꝛke is doone, but I doo not perceaue howe $\frac{15}{60}$ is greater then $\frac{1}{4}$. For yf I ſhall vſe my former maner of examina tion, by the partes of ſomme coyne, I ſee that $\frac{3}{5}$ of a crowne, is 36 d, and $\frac{5}{12}$ of a crowne is 25 d, wherof the one multiplied by the other, doth make 900 d, whyche is 15 crounes, but by your multiplication ther amounteth $\frac{15}{60}$ whiche is but 15 d, and that is leſſer muche then any of bothe the fyrſte fractions. Mai. That d. fference is betweene multiplication in hole numbꝛes, and multi plication in bꝛoke numbꝛes, that in hole nũ bꝛes the ſomme that amounteth is greater then both the other whereof it came, but in fractions it is contrary waies, for the ſum that amounteth is leſſer then any of the other two fractions, wherof it came. Stu. I beyng muche to vnderſtande the reaſon therof.

Mayster. Although I purpoſed to reſerue the reaſons of woorkes Arithmeticall for the perfecte boke of Arithmetike, yet I will ſhewe you this, bycauſe of the ſtraungenes of the woorke. You ſee in whole numbres, that of two numbres beyng multiplied together is made the thyrde numbre, whyche thyrde numbre doth beare the ſame proporcion to the numbre multiplied, that the multiplier doothe beare to an vnitie. And ſo in fractions the thyrde numbre whiche amounteth of multiplication, beareth the ſame proporcion to eche of the two fyrſte fractions, that the other of thoſe two fractions dooth beare to an vnitie. Scholar. Syr I vnderſtande youre woordes thus: When 40 is multiplied by 12, there doothe amount 480, whyche 480 doothe contayne 40 ſo many tymes in it, as 12 dothe conteyne vnities, that is to ſay, xii. tymes. And ſo it appereth that 480 doothe conteyne 12 ſo manny tymes alſo, as 40 doothe contayne vnitees, that is forty tymes, but now I ſee not how the thyrde numbre in this example of fractions can conteyne any of the two former (as it happened in hole numbres) ſeynge it is leſſer then eyther of them. Mayſter. No mervayle, yf you can not ſe that thyng, whi che is not poſſible to beſene of any manne, howe the thyrde numbre in multiplication of fractions ſhoulde bee greater than any of

the two former fractions, but yet this maye you see (whiche I sayd) that the thyrd number in fractions so multiplied doothe beare the same proportiõ to any of the two former fractions, that the other of those .ij. fractions dooth beare to an vnitie, as in your example $\frac{3}{5}$ beyng multiplied by $\frac{5}{12}$ doth make $\frac{15}{60}$. Nowe say I, that $\frac{15}{60}$ doothe beare the same proportion to $\frac{3}{5}$, that $\frac{5}{12}$ doothe beare to an vnitie, as you may in your owne forme of examination by coyne trye it, for in an olde angel are 180 halfepence, whiche I sette for the intiere vnitie, whose partes accordynge to the fractions aforesayde are these, for $\frac{15}{60}$ sette 45 ob. for $\frac{3}{5}$ take 108 ob. and for $\frac{5}{12}$ put 75 ob. Nowe doothe 45 beare the same proportion to 108, & 75 doth beare to 180. for 45 is $\frac{5}{12}$ of 108, and so is 75 also $\frac{5}{12}$ of 180. But these resons may be better reserued tylle an other tyme, when the knowledge of proportions in due order shall be taughte, yet in the meane season I wyll shewe you how it cometh to passe, that in fractions the thyrde summe multe nedes be lesser then any of the other two. Consyder thus, that when a fraction is proponed as in the former example $\frac{3}{5}$, if it be multiplied by more then one, it wyll make more than one intiere numbre, As yf I multiply this $\frac{3}{5}$ by 5, that is to saye, yf I take it fyue tymes, it wyll make three intiere

vnities, exaumple in a crowne, $\frac{1}{3}$ of it maketh 3 s, whyche yf I take fyue tymes, it will amounte to 15 s. that is three intiere crounes: so yf I take the same $\frac{1}{3}$ but twise, it wyll yelde 6 s, that is, one intiere crowne and $\frac{1}{3}$. Nowe yf I take it but ones, it can be no more than it was before, that it is 3 s. And yf I take it lesse than ones, it can not be so muche as it was before. Then seeyng that a fraction is lesse then one, yf I multiply a fraction by an other fraction, it foloweth, that I dooe take that fyrste fraction lesse then ones, and therefore the summe that amounteth muste needes bee lesse then the fyrste fraction. Scholar. Syr I thanke you muche for this reason, And I truste, I doo perceiue the thynge, as by exaumple of this same fraction $\frac{1}{3}$, I wyll expresse: yf I take $\frac{1}{3}$ of a crowne ones, that is to saye, yf I multyplie $\frac{1}{3}$ by 1, it wyll be, as it was before, but 3 s, so yf I dooe multiplye it by $\frac{1}{2}$, that ys; yf I take it but halfe one tyme, thanne wyll it bee but halfe so muche: lykewayes, yf I multyplye it by $\frac{1}{12}$, that is, yf I take it but the thyrd part of ones, yt wyll yelde but 12 pence, that ys the thyrde parte of the fyrste fraction. And so to make an ende, yf I take yt but the twelfthe parte of ones, that is, yf I dooe multiplye it by $\frac{1}{12}$ yt wylle yelde but

but the twelfthe parte of the fyrſt fraction, whyche is but 3 pence. And ſo it folowed that yf $\frac{1}{12}$ make 3 pence, than $\frac{6}{12}$ muſte needes make fyue tymes ſo muche, that is 15 pence, whych was the ſumme, that hath gyuen the occaſyon of all this doubte.

Mayſter. Thenne I perceaue you haue ſufficiente vnderſtandynge in this ſorte of Multiplycation for this tyme, wherefore I wyll omytte that I myght ſaye more of Multiplycation, tyll we comme to Reduction, and wyll paſſe to the other woorkes, and fyrſte to Dyuyſyon, whoſe place folowed Multyplycation, bothe by natucall order, and alſo in eaſyneſſe of woorke.

DIVISION.

When ſo euer twoo Fractions be yꝺponed, that the one ſhoulde bee diuyded by the other, I muſte ſette downe fyrſt the fraction that ſhall bee dyuyded (whyche is called the diuydende) and thenne after yt the other, whych is the dyuiſour. Then ſhall I multyplye the numeratour of the diuidend by the denominatour of the diuiſor, and that whych amounteth, I muſt put for a newe numeratour: agayne I ſhall multiply

the denominatour of the dividende by the numeratour of the diuisour, and the numbre that amounteth thereof, I must put for the newe denominatour. And this thyrde fraction is the quotient of the sayd diuision.

Scholar. This seemeth easyer in fourme, as by exaumple thus: If I woulde diuide $\frac{5}{6}$ by $\frac{2}{3}$, fyrst I multiply 5 (beyng the numeratour of the dividend by 6, whiche is the denominatour of the diuisour, and thereof ryseth 30, then I multiply 3 (beyng the denominatour of the dividende) by 2, beynge numeratour in the diuisor, and so ryseth 16, the whyche I muste make in a thyrde fraction thus $\frac{30}{16}$. Maist. Me seemeth you are quycker in vnderstandyng now, thenne you were, whenne I taught you the arte of hole numbres, but that is no meruayle, for the more knowledge that any man getteth, the redyer shall he fynde his wyt, and quicker in vnderstandyng, but yet of ii. thynges I wyll admonyshe you, whiche you myght haue obserued here for ease of woorke and lyghtnesse of vnderstandynge the nature of the quotient.

When soeuer you do diuide one fraction by an other, other they bee bothe equall togyther, other elles the one is greatter thenne the other, yf they bee equalle, theyr quotiente shall be suche, that the Numeratour and the denominatour of it shall be equal al

so.

so. And if the two first fractions be vnequall their quotient shall declare the same, by the vnequalitie of the numerator and denominator, as in these examples folowyng, shall apeare. Fyrst of equal fractions, $\frac{4}{5}$ and $\frac{12}{15}$ be equal togither: and yf the one be dyuyded by the other, the quotient wyll bee $\frac{300}{300}$, as you may perceaue by ỹ rule aforesayd. Now in vnequall fractions, as $\frac{4}{5}$ and $\frac{3}{10}$, the quotient will be $\frac{40}{15}$: wher the numerator is greater then the denominator. S. I see it as so, but I se not the reason why it should bee so. M. The reason is thys. When any fraction is dyuyded by an other, the quotient declareth what proportion the dyuidend beareth to the diuisor. And note this, that the numerator in the quotient representeth the diuidend, and the denominator representeth the diuisor. And this is alwayes true, whether the greater fraction be dyuyded by the lesser or the lesser by the greater. But thys proportion will not be exactly knowē, tyll you haue learned the arte of proportions, notwithstandyng somwhat of it wyl I declare in ỹ nexte rule of Reduction. But now for the easy remembraunce of the quotient in diuision, as soone as you haue set downe your two fractions, the one agaynst the other, then make a lyne for the quotient: and as soone as you haue multiplyed the numerator of the dyuidend, by the denominator of the diuisor,

ber the numbre that amounteth ouer the laſt lyne, & the multiply the other twoo numbres & ſet their total vnder the ſame lyne. S. I per ceue, you wold not haue me truſt to memory tyl I were better expert, leaſt often tymes I happe by myſſe remembrance to be abuſed. For this example I take for declaratiõ, yf I wold diuide $\frac{2}{3}$ by $\frac{4}{5}$, I muſt ſet the numbres one agayuſt the other (as here both appeare) & then make an other lyne for the quotiẽt in ſom good diſtãce where I may ſet the numbres of the quotient, as ſoone as any of them is multiplyed: ſo then as ſoone as I haue multiplyed 2 by 4, which maketh 8, I ſhall ſette that 8 ouer that lyne, thus $\frac{8}{\ }$, and then multiply 3 by 3 which yeldeth 9, and that 9 muſt I ſet vnder the ſame lyne, and then wyll the whole quotient appeare thus, $\frac{8}{9}$. wherby appereth (as I remembre your woordes) that $\frac{2}{3}$ is in proportion to $\frac{4}{5}$ as 8 is to 9. but how maye I perceaue that? M. Although you ſhall better perceaue it by the rule of Reduction, yet this example may be declared in cõmon coyne, as in a cõmon ſhylyng of xii. pens, of which $\frac{2}{3}$ maketh 8 d, & $\frac{4}{5}$ doth make 9 pens, and ſo you may eaſyly ſee, that their proportions do agree. And if you had taken this example before, when you tooke the example of $\frac{4}{5}$ and $\frac{2}{3}$, your quotient wold apeare as this booth, more eaſyer to vnderſtand, where as

2	3
3	4

that

that quotiēt beyng $\frac{3}{4}\frac{0}{5}$, is not an easy proportion for you to perceaue, beig yet little acquainted with proportions: whereof to giue you some taste, I wyll enter to the rule of Reduction, in which also I wyll declare o˜ her workes both of Multyplycation, and also of Diuisiō, which now I must for a time omit, as things that do need the help of Reduction.

REDVCTION.

THerefore wil I now declare the diuersities of Reduction of fractiōs, wherein mōly hath v. varieties. First when ther be sundry fractiōs of one intier vnite, ther must be reduced to one denominacion, & also into one fraction. Secondarily, when ther be proponed fractions of fractions, they must be reduced likewise into one fractiō, for otherwaies, thei cā not be brought into one denominacion. Thyrdly, whē an improper fraction is proponed, that is to say, a fraction in form, which in deed is greater thē an vnite, it must be reduced into apt form, expressing the vnite or vnites of it, & the proper fractiō distictly. And some times also it shal be nedeful to cōuert such a mixt nūber of vnites & fractiōs, into ỹ form of a fractiō, ỹ is, into an improper fractiō, which 2 formes I esteeme but as one, bicause thei work on one kind of nūber. Forthly ther happeneth sometymes fractions to be wrytten in great numbers, whych might be written in lesser numbers, therefore is there a meane to reduce suche

5 varieties of Reduction.

1

2

3

4

greate numbres into theyr smalest rearmes. Fyftly, whan any fraction betokeneth the partes of a whole thyng, which hath by cõmon partition certayne partes, but none of lyke denominacion with that fraction, then maye you reduce the sayde fraction into an other, whose denominacion shal expres the cõmon partes of that whole thyng. S. This distinction in doctrine delyteth me muche, but more with hope then present frute, for as yet I doo not vnderstand scarcely the varieties, and muche lesse the practise and vse of their workes. M. Reductiõ is an orderly alteracion of numbres out of one forme into an other, whiche is neuer doone orderlye but for some needfull vse, as in euerye of the said 5. seuerall varieties I wyll distinctlye declare. Fyrste therefore, when two, or more seuerall fractions of any vnite bee proponed as for example, $\frac{3}{12}$ and $\frac{4}{5}$: bycause it is harde to tell what portion of the intiere numbre these ii. fractions doo expresse, therfore was reduction diuised, to bee a meane whereby these seuerall fractions myght bee brought into one denominacion and fraccion. And in these fractions thys is the arte for bryngyng them to one denominacion. Multiply fyrst the denominatours togither and the totall therof you shal set twyse down vnder ii. seuerall lynes, for two newe denominatours, or rather for one common deno-

The fyrst reason of reduction.

mina-

minatour: Then multiply the numeratour of the fyrſt fraction, by the denominatour of the ſecond, and ſet the total therof for the numeratour ouer ỹ fyrſt lyne. Likewyſe multiply the numeratour of the ſecond fractiõ by the denominatour of the fyrſt, and ſet that total ouer the ſecond lyne for the numeratour of that fraction, and ſo are thoſe two fyrſte fractions of ſeueral denominaciõs brought to one denomination. S. If I vnderſtande you, as I thynke I do, my example ſhal declare the ſame. The fractions which you proponed were theſe, $\frac{1}{16}$ and $\frac{4}{6}$, whoſe denomynatours (beyng 16 and 6) I multiplye togyther, and there amounteth 96, whych I ſet vnder ii. lynes, thus. $\frac{\quad}{96}$ $\frac{\quad}{96}$, then I multyply the numeratour of the fyrſt fraction, by the denommatour of the ſeconde, ſayng, 3 into 6, maketh 18, ỹ ſet I ouer the fyrſt lyne for a new numeratour, and it wyll be thus $\frac{18}{96}$ lyke wayes I multiplye the numeratour of the ſecond fraction by the denominatour of the fyrſt, ſayinge: 4. tymes 16, maketh 64. that I ſet for the ſecond numeratour, and the fraction wyll appeare thus $\frac{64}{96}$, ſo that both fractions brought to one denomination, muſt ſtand thus $\frac{18}{96}$ and $\frac{64}{96}$. M. You haue doon well. Scho. I beſeeche you, lette me exampne yt after my accuſtomed fourme, by common partes of coyne. M. Be it.

§. A newe angell accompted at viii. ſhyllyngs

ges con

ges containeth 96 pens, wherof $\frac{1}{16}$ that is the xvi. part, is vi. pens, and $\frac{3}{16}$ is 18 pens, that is $\frac{18}{96}$. Again $\frac{1}{6}$ of the same Aungel, is 16 pens, so that $\frac{4}{6}$ maketh 64 pens, that is $\frac{64}{96}$. And so I find the summes to agree with thother before. M. So haue you now the art to bring such 3 fractions into one denominacion. And if ther be more then 3, then must you multiply all the denominators together, & set the total therof so many tymes downe as ther be fractions, and then to get for the one a new numerator, multiply the numerator of the fyrste, by the denominator of the seconde: and the totall therof multiply by the denominator of the thyrd, and so forth yf there be more. Lykewise multyply the numerator of the second, by the denominator of the fyrst, and the totall therof by the denominator of the thyrd. And in the same sorte multiply the numeratour of the thyrd into the denominator of the first, & the totall therof into the denominatour of the second, & so forth if ther wer moe. So these 3 fractiõs $\frac{2}{3}$ $\frac{3}{4}$ $\frac{5}{6}$ doth make by reductiõ these other 3 fractions of one denominacion, $\frac{240}{360}$ $\frac{270}{360}$ $\frac{288}{360}$. All whych you may bring into one fraction by addyng the numeratours togyther, and puttyng that totall for the commõ numerator, reseruyng stil that same cõmon denominator. And those 3 fractiõs make one. simplyce fractiõ thus $\frac{798}{360}$. S. All this I per-

ceaue

reaue, & also that this laste fraction is more then an vnyte, and therfore you dyd call it an improper fraction. M. There be certain other formes of worckyng in thys reductiō, whych I wyll briefly touch also, to gyue you an occasion to exercise your wit therin. The fyrst varietie ys thys, whē you haue made & wrytten down your common denominator (as I haue taught before) thē to get a numerator for the fyrst, do thus. Deuide the common denominator by the denominator of ỹ fyrst fraction, and the quotient multiplyed by the numerator of ỹ same, yeldeth a newe numerator for the fyrst new fractō, so lykewyse do wyth the second and the thyrde, and wyth all the residue, yf there be more &c. The fyrst varietie of this reduction.

That wyl I proue in your laste example of these 3 fractions $\frac{2}{5}$ $\frac{3}{4}$ $\frac{2}{3}$. when the denominators be multiplied, they make 60. for 5 in to 4 maketh 20 and 20 by 3 yeldeth 60, that I sette downe 3 tymes, thus 60. 60. 60. thē to haue a numerator for the fyrst, I must deuide 60 by 5, (the denominator of the first) & the quotient is 12, whych I must multiply by 2 (the numerator of ỹ fyrst, and $\frac{24}{60}$ maketh 24, & so haue I for the fyrst fractiō $\frac{24}{60}$. lykewyse for ỹ second fractyō I dyuyde 60 by 4. & ther cometh 15, whych I multyply by 3, & so haue I 45, & the secōd fractiō $\frac{45}{60}$. The for ỹ third in like sort wil com $\frac{40}{60}$. M. Another way is this. If it happē so ỹ the lesser denominator

The seconde varietie.

minatour can by anye multiplication make the greater, thē note the multiplyer, and by it multiply the numeratour ouer that lesser denominatour, and for the lesser denominatour put the greater, as thus in these two fractions $\frac{3}{12}:\frac{2}{4}$, 3 beyng the lesser denominatour multiplyed by 4 wyll make 12 whych is the greater denominatour, therfore by the same 4 I do multyply 2 whyche is numeratour ouer 3, and that maketh 8, vnder whyche I put 12 being the greater denominatour, whyche is also made by multiplycation of 4 into 3, and so haue I these twoo fractions $\frac{3}{12}:\frac{8}{12}$ thus shortly reduced without alterryng the one fraction. Sco. Thys I vnderstand. M. Then marke thys thyrde waye: yf the denominatours do not happen so, that one by multiplycation may make the other, then looke whether they both may be partes of any other one numbre, as in $\frac{9}{12}$ and $\frac{7}{18}$ although the lesser taken but twyse bee to great to make 18, yet they both may be partes vnto 36, therfore loke how many times 12 is in 36, and that quotient beyng multyplyed by the numeratour ouer 12, the totall shal be put in steed of $\frac{9}{12}$ numeratour ouer 12, and for 12 put 36, thus $\frac{27}{36}$. So lyke wayse loke how often is 18 in 36, and bycause it is twyse, therfore by 2 multiply 7, whyche is ouer 18, and it wyll be 14, set that for the numeratour, and in steed of 18, put 36, then

shall

shall your fractions reduced stande thus $\frac{2}{3}$ $\frac{3}{6}$ in steede of $\frac{1}{2}$ and $\frac{7}{15}$. And if you will proue, whether you haue wrought well or no, that may be proued by reduction of them agayn to theyr former denominations, wch the arte shall bee taught in the fourth kynd of reduction, where greater termes of fractions bee reduced into smaller in numbre, but not smaller in proportion. And if in suche Reduction the same termes or numbres come agayne that were before, than is the woorke good, elles not Scho. Syr I heare your woordes, but I doue not vnderstande many of them, whyche it may please you to declare. Mai. With a good wyll, whan conuenient place serueth, but that must be in the sayd.iiii. kynde of reduction. In the meane season I wyll declare the second forme of reduction, whiche teacheth how to reduce fractions of fractions into one fraction, and so to one denomination When fractions of fractions be proponed, you shal multiply the numerators of eche into other, and set the totall for the newe numerator, than multyplye all the denominatours lyke wayes, and take their totall for the new denominator, and so are they spedyly reduced. Sco. If that be all, then I vnderstande it all ready, as by this example I wyll declare. These bee the fractions, $\frac{1}{4}$ $\frac{2}{3}$ $\frac{6}{7}$ $\frac{7}{9}$, whyche I woulde reduce to one denomination.

Redu
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Therfore begyn I with the numeratoures, and multiply them altogyther, saiyng, 3 into 2 maketh 6, and 6 by 6 maketh 36, whi che multiplied by 7 yeldeth 252, that I set ouer a lyne for the numeratour, thus: $\frac{252}{}$ Then I multiplie the denominatours 4 by 3, maketh 12, and that by 7 bryngeth 84, whyche multiplied by 9, yeldeth 756, the newe denominatour. And so the whole reduced fraction is this $\frac{252}{756}$ whyche is to harde a fraction for me to vnderstande yet, Maister. You thynke so, and no meruaile, but anon you shall learne to iudge it easily. for this fraction is no more in dede then $\frac{1}{3}$, although it be in greater termes, and therefore more straunger and more obscure. And this suffiseth for this Reduction, saue that I will shew you by a figure of measure the iuste rate and reasone of this kynde of fractions, and also the newe vnderstandyng of their Reduction.

The intiere measure parted into 9.

1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	$\frac{7}{9}$	
.1.		.2.		.3.		$\frac{6}{7}$		
1	2	3	4	$\frac{2}{3}$				
1	2	3	$\frac{3}{4}$					

Here you see the longest measure, whyche standeth for the hole and intiere quantitie fyrste parted into 9 diuisions, whereof 7 are seuered

seuered by the seconde measure, and thereof agayne are parted out 6. And that 6 beyng diuidyde into 3 partes, 2 of theim are parted by the fourth measure, of which fourth measure beyng diuided into 4 partes, the lowest measure doothe conteyne $\frac{1}{4}$, so that the same $\frac{2}{4}$ muste bee truely named, not $\frac{2}{4}$ of the hole measure, but in deede is $\frac{2}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$, or as I wolde rather expresse it, $\frac{2}{4}\frac{2}{3}\frac{6}{7}\frac{7}{9}$. Scho. This example is so sensible, that I can not choose but see it. And farthermore I see also, that the same fraction is equall to $\frac{2}{3}$ of the intiere measure, as the lynes whyche runne vp and downe doo expressely set foorthe. Also I see here, that $\frac{2}{3}\frac{6}{7}\frac{7}{9}$ is equall to $\frac{4}{5}$. And farther yet that $\frac{6}{7}\frac{7}{9}$ is equall to $\frac{6}{9}$. Ma. I am gladde that you see it so welle, not doubtynge but you wyll gather greater lyghte of knowelege hereby. But nowe it is tyme that we come to the thyrd forme of Reduction, whyche treateth of improper fractions, that is to say, myxte numbers of vnites and fractions, although they appeare like fractions, as this $\frac{7}{5}$ whyche dooth include 5 vnites wholy, and $\frac{1}{5}$ ouer. Wherfore fyrst you shal know them, by the numerator is greater than the denominator. S. In deede sir that appeareth reasonable, that if the numerator doo expresse more partes to be taken of any vnite, than the denominator doth signifi that

The thyrd forme of Reduction.

vnite to be diuided into, it must nedes folow that suche a fraction improueth more than the hole, that is to say, the hole with certain partes ouer. But what reduction is there in it? M. There be .ij. seuerall kyndes of reduction, concernyng suche fractions. Some tymes it shall bee nedefull to conuerte thole fractions into the vnites and the proper fraction that will remayn. And sometymes contrary wayes, it shall be mete to reduce myxte numbres, that is vnites writen with fractions into the forme of one symple fraction, so be there .ij. wayes. Sco. What is the meane of the fyrste waye, to tourne improper fractiõs into vnites with their proper fractiõs? Maist. That is thus. Your numerator beynge greater then the denominatour, muste be diuided by the same denominatour, and the quotient therof expresseth the vnites, the remayner shall bee put for the numeratour of the fraction that resteth, and the denominatour muste be the same that was before Scholar. For example, I take $\frac{17}{5}$. And diuidyng 17 by 5, the quotient wyl be 3, and ther wil remain 2. M. That must you write thus. $3\frac{2}{5}$ where (you see) I haue writen 3 without any lyne, as vnites numbres ought to be written, and the 2 that remayned, I haue set ouer the former denominator with a lyne, as a proper fraction. And this numbre doth signifye nowe 3 vnites, and $\frac{2}{5}$ of one.

one. Scho. Then yf I wold by vnites here vnderstand crownes, for it were 3 crownes, and ¼, that is 2 s. M. Euen so, & therefore ¼ dyd signifie the same. But this happeneth somtymes, that whan the reduction is so wrought, there remayneth nothyng. And then is it not a myxte numbre, but a symple intiere numbre represented lyke a fraction. Scholar. As ¼ wylle make 3 iuste, and ⅛ wylle make euen 6. This I wyll remembre. But nowe what is the seconde fourme of reduction that you spake of for these sortes of fractions? Mayster. When so euer you haue any of these two sortes of numbres, that is to say, hole numbres without fractions, or whole numbres with fractions, and you wolde fourme them into the fourme of a fraction, you muste multiplye the whole noumbre by that denominator, wyche you wyll haue to remayne styll, and to the totall therof adde the numerator, wyche you haue all readye, and all that shall you sette for the newe numerator, kepyng styll the former denominatour, as if you haue 6 ¼, wyche you wolde conuert into an improper fraction, you muste multiplye 6 by 4, wherof commeth 24, and therto adde the numeratour, wyche is 3, and so haue you 27 for the numeratour, and 4 styll for the denominatour. Scholar. Then is ²⁷/₄ equall to 6 ¼.

Mayster. Euen iuste, and so backewarde (as appeareth by the former Reduction) 6. $\frac{3}{4}$ maketh $\frac{27}{4}$, and thus one of these reductions may be the proofe of the others woorke. Scholar. This I perceaue, but now if you would tourne whole noumbres without fractions, into any fraction, I see not howe that may bee done, bycause there is no denominatour to make the multiplication by. Mai. That was wel marked, but this you knowe, that no man intendeth to tourne any whole numbre into a fraction, but he hath in his mynde that denominatour, by whiche the multiplication must bee made: for the proofe wherof, I set downe 7, whiche is a whole numbre. And if you will haue this nũbre conuerted into any certaine fraction, wyll me to doo it. Schol. I praye you reduce 7 into a fraction. Maist. Then you care not what the fraction bee, so it bee some fraction. S. No I passe not for the sort of the fraction. Mai. Then howe can you thynk, that you requyre me to do any thyng certayne, whan you leaue me to doo as I lyst? And seeynge you stande at that staye, whether thynke you that I muste fyxte intende in mynde, what fraction I wyll make of it, before I can doo it in deede? Schol. Elles you shoulde doo ignorauntly. Maist. Then I wyll lymytte my selfe (seeynge you wyll not) to tourne it into quarters. And ther-

therefore I multyplye 7 by 4 (whyche is the denomynation of quarters) and there amounteth 28 to bee sette for the numerator, and the 4 muste bee sette for the denominatour, and the fraction wyll bee thus $3\frac{8}{4}$. Scholar. In deede I perceaue this to bee reasonable, for without muche tryall I vnderstande, that $\frac{2}{4}$ of any thyng, doeth make 7. And so then if I would turne 8 into fyfte partes, it wyll make $4\frac{8}{5}$, which is all one with eyghte. for 8 crownes, tourned into fyfte partes (that is into shillyngs) will make 40 shillings, that is $\frac{4}{5}$ of a croune Maist. Seyng you vnderstande nowe theise three kyndes of Reduction, I wyll declare vnto you the fourthe kynde, that is when fractions bee written in greater termes then they nede, howe they maye bee broughte to lesser termes. Scholar. To wryte any fraction in greater termes thanne needeth, seemeth to be a fault, and so this rule seemeth to amende that faulte. Mayster. It were a faulte to doo any thynge without nede, whyche after muste bee redressed, but in this case it is not so, nother dyd I say absolutely (as you dooe) that it needeth not, to expresse those fractions in so greate termes, but that the Fractions dooe not neede, I meane for their valewe to bee vnderstande, but yet it maye bee needefull for the ease of those woorkes whereto they bee applyed,

The fo kynde reductio

as for example, in the first kynde of reduction, this was youre owne example: $\frac{1}{16}$ and $\frac{4}{6}$, whyche whanne you woulde reduce, you were fayne to tourne theym first into one denompnation, and so appeared they thus, $\frac{18}{96}$ and $\frac{64}{96}$ whete the fractions (for they owne vnderstandynge) needed not to bee tourned out of smaller tearmes into greatter, but yet the easynesse of woorkyng needed it. Scholar By I vnderstande nowe not onely the dyfference of this neede (for the fractions myghte better bee vnderstande, as fractions seuerall, eche in his valewe, whenne they were in lesser tearmes, although they coulde not so well bee reduced) but also I vnderstande what you meane by greater termes and lesser termes, whereof before I was in doubt, for I see you calle the Numerator and Denominator the termes of the fraction. Maister I am gladdo you vnderstande it so well. Nowe than whenne you woulde valewe any fractions, bycause that may bestt bee doone, whenne the tearmes are moste smallest, you shall reduce theym to the smallest that you cannc, whych thyng you may doo thus: Dyuide the greatest of any twoe two termes by the lesser, and yf any thynge remayne, by that remayner diuide the laste diuisour, and if any thyng remayne now, by that diuyde the laste diuisour,

termes of fractions.

soure (whiche was before the remayner of the first diuision) and so continue styll, tyl no thyng do remayne in the diuision, and then marke youre laste diuisour, for it is the noumbre that wyll easyly reduce your fraction, if you diuide both the numerator & the denominatour by the same noumbre, and putte for the numeratour the quotiente of his diuision, and for the denominatour also his quotiente, that ryseth by his diuision. S. I take for example $\frac{18}{96}$, and bycause 96, is the greater noumbre, I diuide yt by 18, and the quotiente is 5, and there resteth 6, what shall I do with this quotient? M. Nothyng in thys woorke. but nowe seynge there remayneth somewhat, by that remayner must you diuide the laste diuisor. S. If I shall diuide 18 (whiche was the laste diuisour) by 6 that wil is the remayner so is the quotient 3, and nothing resteth. M. As for the quotient omyt hym yet, but bycause there dooth remayne nothing therefore is syre (whyche was your laste diuisor) that numbre, by whyche you maye reduce the fraction proponed. S. Then as you taughte me, I must diuide the numerator 18 by 6, & the quotiet is 3, which I must put for ỹ numerator ouer a lyne, thus $\frac{3}{\ }$ and then by the sayde 6, muste I diuyde also the denominatoure 96 and the quotiente wyll be 16, whych I must take for thy deno-

minator, and so is the fraction $\frac{3}{6}$. And so me thinketh this rule doth proue the worke of the first reduction. M. That is true, if the first reduction were made of fractions in their least termes, and els not without some helpe, as the seconde numbre in that place wyll declare. S. The second numbre was $\frac{4}{6}$, whyche was tourned into $\frac{64}{96}$, by that rule. Now yf I shall by thys rule reduce it again into the least tearmes, I must diuide 96 by 64, and there resteth 32, by whych 32 I diuide 64 and there remayneth nothing, wherefore I must take that 32 for the diuisor to reduce the layd fraction. Then do I diuide 64 by 32, and the quotient is 2, which I sette for my numerator. Againe I deuyde 96 by 32, and the quotient will be 3, and so haue I $\frac{2}{3}$ but. M. Muse not at the matter, for you haue done wel inough: but you think you haue not the fraction that you loked for, that is $\frac{4}{6}$, yet haue you one equall to it, as by the partes of a shyllyng you maye proue. S. Truth it is, for eyther of them wyll bringe forth 8 pence, so that $\frac{8}{12}$ and $\frac{4}{6}$ and $\frac{2}{3}$, be all three equall. And nowe I perceaue that because $\frac{2}{3}$ was not written in the least tearmes that it myght bee, therefore thys reduction brought forth not it, but that other whyche is written in the least tearmes. Nowe vnderstand I thys rule well. But is there anye other way to worke thys reduction? M. Yes but

but fyrst note this, that if you find no such dy
uisor to reduce the fraction tyll you come to
1, bycause 1 doth make no diuision, therefore
that fraction is already in his least tearmes
as $\frac{2}{1000}$ you may proue. and so of $\frac{3}{565}$ & ma
ny other lyke. But now to your question, yf
you can without that diuision, by memorye
espye the greatest numbre that may dyuide
exactly both tearmes of your fraction pro-
poned, then need you not to vse that diuisio
as in this fraction $\frac{6}{900}$, I see that 1, 2, 3, 6 be the
greatest numbre that can dyuide them both,
and therfore without any worke by memorye
only, I turne that into $\frac{1}{150}$, but thys habilitye
in knowledg, is gotten by exercise. Yet one
other way of easye reduction in thys kynde
ther is, when your fraction hath any cyphers
in the fyrst places of bothe termes, for then
may you by casting away the cyphers, make
a fyrer reduction as thus $\frac{3000}{4000}$, here take a-
way the cyphers, and it wyll be $\frac{3}{4}$, whych is
the same in valew with $\frac{300}{400}$. S. And so if I
haue $\frac{4000}{5000}$ yt wyll be $\frac{4}{5}$. M. You are decey-
ued, for you take away more cyphers frō the
numerator, than you do take frome the de-
nominator, whych you may not doo. S. I
confesse my faulte, which came of to muche
haste, I was more gladder of the rule, then
wyse in vsynge it, but now I vnderstand it,
I truste. M. Then may I go in hande wyth
the fyfte or laste kynde of Reduction, which

The f
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teacheth

teacheth how to tourne any fraction proposed into any other denominacion that you lyste, or into any partes of common coyne, weightes or meiures, or such like. For declaracion whereof, lyst you that marke whether your fraction be a symple fraction, other els a fraction of sundry parts, I meane of more fractures than two. And if your fraction be a fraction of fractions, or otherwayes compounde, you must reduce it to one symple fraction. And then marke well the denominacio of that other fraction into whych you wold tourne thys, for by that denominatour you muste multiplye the numeratour of youre fyrste fraction, and the totall thereof shall you dyuide by the denominatour of youre fyrste fraction, and that quotient shall be the numeratour to the denominatour proposed: as for example I haue this fraction $\frac{1}{2}$ whych I wolde tourne into tenthe partes, therefore I multiplye this 1 by 5, that is the numeratour of my fraction, and there ryseth 5, whych I diuide by 5, and the quotient is 1, which must be the numeratoure to 10, and so $\frac{1}{2}$ wyll be $\frac{5}{10}$. 5. Thys is easy ynough to do. M. Then shall you se an other example of the same fraction, that is not so easy, as if I wolde tourne $\frac{1}{2}$ into viij partes. proue you that wo[illegible]ke. Sc. I muste multiply 8 by 3, and there amounteth 24, whych I dyuide by 5, and the quotient

is 4, then is the newe fraction $\frac{4}{7}$ M. And see you nothyng doubtfull in this worke? S. I see that when 24 was diuided by 5, there remayned 4, which I dyd not passe of, bycause you spake nothyng of any remayner, but only of the quotient. M. Wherfore you remembre what I sayd to you in dyuision of whole numbres, that you shoulde not passe of the remayner there, but onely note it, as a summe that coulde not be diuided without knowledge of fractions. Wherfore now marke thys, that in all diuisions of whole noumbres, when there is anye remayner, you shall sette it ouer a lyne as a numeratour, and set the diuisor for the denominatour, and that fraction dooth make the diuision complete, and is part of the quotient. And if I wold diuide 48 by 5, the quotient wyll be 9 $\frac{4}{5}$, so in your former woorke when 24 was diuided by 5, the quotiente shoulde bee 4 $\frac{4}{5}$, and so the newe fraction shoulde bee $\frac{4}{\frac{4}{5}}$ that is $\frac{4}{7}$ of the entiere numbre, and $\frac{4}{5}$ $\frac{1}{5}$, whyche you maye proone by exaumple of some coyne, Scholler. Then I take a crolwne whose $\frac{4}{5}$ is 3 $\frac{1}{5}$. Nowe yf I woulde proone whether 3 $\frac{1}{5}$ bee $\frac{4}{\frac{4}{5}}$ I shall haue a combrouse worke to doo. M. In deed for whole pennyes your exãple is troublesome, yet turnyng the crown into halfpennyes, it is easye ynough.

Scol-

S. Now wyll I do it. M. Fyrst let me tel you an ealy way how to fynde any numbre, that wyll ealyly be diuided into suche partes as you desyre, whych way is thys. Set downe the partes that you desyre, and then by one of them multiply all the other, the totall whereof shall contayne all the partes proponed, as if I woulde haue a numbre that may be dyuided into 4 5 6 and 7 partes. by 4 multiplyes, and ther ryseth 20, then multiply 20 by 6, and it wyll make 120, whyche multiplyed by 7, wyll yelde 840, and so of any other numbres. S. Then in our former example where is mention but of 5 partes & 8 partes, I shall only multyplye 5 by 8, why che maketh 40, and that numbre wyl serue M. So wyl it. S. Then what is $\frac{3}{5}$ of 40? M. Proue by the same rule whych you confesse ealy inough. 3 tymes 40, is 120, whych beyng diuided by 5, maketh 24. and $\frac{4\frac{4}{5}}{8}$ is iuste. Nowe to knowe whether $\frac{4\frac{4}{5}}{8}$ be equall to 24, fyrst I see (by the same rule that $\frac{4}{5}$ is 20) and $\frac{1}{8}$ is 5, of whych 5 I must take $\frac{4}{5}$ and that by the same rule is 4. So that I see nowe, that $\frac{4\frac{4}{5}}{8}$ is equall to $\frac{3}{5}$. M. And by the way note this forme of fraction, how it is wrytten, that is to say, both the numeratour and his fractiō aboue the lyne. Although I know it may be wrytten other wayes as thus $\frac{4}{5}$ and $\frac{4}{5}\ \frac{1}{8}$ but I accompte the other waye more apte

a great

a great deale. And sometyme you expresse by an other waye then aforesayde mentioned all fractions of fractions, as thus. $\frac{3}{4}\frac{5}{8}$ that is $\frac{3}{4}$ of $\frac{5}{8}$, and so of other. But I tempt these formes to the arbitriment of euery wyse artes manne, to vse as he thynketh moste apte and ready. But nowe one example more for thys rule, and then shal we ende it, if I haue $\frac{7}{15}$ of a soueraygne (accomptyng the soueraygn 20 shyllynges) howe many shyllynges is that $\frac{7}{15}$? S. I muste multiplye 7 by 80, and that maketh 140, whiche I shall deuyde by 15, and the quotient wyll be $9\frac{4}{15}$. M. That is 9 $\frac{5}{5}$ and $\frac{4}{15}$ of one shylyng. whyche I woulde haue tourned into pennyes. S. One shylling hath 12 d, therefore sat I thus, 4 times 12 maketh 48, whiche I deuide by 15 and the quotiente is $3\frac{3}{5}$, that is 3d and $\frac{3}{5}$ of a pennye: And if I shall tourne that into farthynges, it wyll be $2\frac{2}{5}$, that is ob.q. almoste, for more exactly it can not be doone with all my connyng. M. No man cannne reduce into more exacter partes then the thyng hathe: Therefore seynge the common partes of 1 d are farthynges, you wente precisely inoughe to accompte $\frac{1}{2}$ farthynge: howbeit it is beste in such case to let the fraction it selfe stand, when you cannot reduce it anye more, for in the fraction can be no errour at all. And this for this tyme shall suffice for reduction, saue that I muste

nowe

Wherin I speke a lytle touchyng Multiplication and Diuision, and so go forward.

MVLTIPLICATION.

In Multiplycation it happeneth sometyme that there be whole numbres to be multiplyed with fractions. And maye be in two sortes, for other the whole numbre is seuered from the fraction, and is the multiplyer, or els, the whole nombre is ioyned with one or bothe of the fractions. and so maketh a mixt numbre therof. If it be in the fyrst sort, then needeth there no reduction, but only multiply the numerator of the fraction by that hole numbre, and the totall therof set for the new numerator, reseruynge styll the former denominator. S. I vnderstande you thus: If I haue $\frac{6}{23}$ to be multiplyed by 16, then must I multiplye that 16 with 6, whiche is the numerator, whereof commeth 96, and that muste I sette for the newe numerator, kepyng styl 23 for the denominator, & so the fractiõ wil be $\frac{96}{23}$ þ is $4\frac{4}{23}$. M. And in this sort of woorke you may abbrege the labour thus: If it happen the denominatour to bee suche a numbre, as may euenly be diuided by the sayde whole noumbre proposed, then diuide it therby, and sette the quotient of that diuision for the former denominator, but sette styll the numerator, and so is the multi-

tipli

but then abatynge 8, it is but 1992, & then last of al, the ¼ is 750, and the 20 more maketh 770, whiche all beeyng added in one summe do make 5280, where the totall sum shulde be but 3000, which summe if you diuide by ¾, so shall you haue ¾ of it, that is 2250, and thereto adde 30 more, then wyll thole 3 summes make 5280, whereby you may see howe this forme as well as the other, doothe declare that the particulars in that questyon wold make more then the whole summe by ¼ and 30 more, and therfore can that question not bee accepted as a possible thyng, but yet doo certayn lerned men propound suche questions, and answere to them. Therfore somwhat to say to their excuse, rather of their good meanyng, then for their dooynge. I will anon declare what may be said for their defēce, but in the meane season I will propound the question as it may bee wrought by good possibilitie. As if. iiii. men buyld a house togither, and it coste them 3000, and then for the partition they agree thus, that as often as the fyrste man dooth pay 6 crownes, so often the second shal pay 4, the third mā 8, and the iiii. mā 3: for els thus, that the first man shal pay double so muche as the iiii. and the seconde man shal pay ¾ of the first mans charge, the thyrd man shall paye double so much as the

3000	
2250	
30	
5280	

seconde: (And these twoo waies are to one end) but farther for their agreement, it is appointed also, that the first man shall geue 6 crounes ouerplus, and the seconde 12, and the iiij. shall geue 20, but the third man shall geue no ouerplus, but shall haue 8 crounes abated of his charge. Nowe is the question possible to be solued, and this is the way to doo it. Marke the proportion of the seuerall charges, and set out small numbres in that rate, by whiche you may reduce the worke to the golden rule, as here in the first forme, the numbres are all redy named, 6, 4, 8, 3, and in the secõd form, although thei be not plainly named, yet thei may be ỹ same nũbres, for 6 is double to 3, & 4 is ⅔ of 6, and again 8 is double to 4. Now adde these togither, & thei make 21, whiche 21 must be set as the first numbre in the golden rule, for if it with the ouerplus of eche mans charge wolde make the totall summe of the charges, then were those seuerall summes the charges of eche man, beside his ouerplus, but nowe it is not so. But yet this is tru, that loke what proportion eche of these seuerall summes doth beare to 21, the same proportion doth the iust charges of euery man (beside his ouerplus) bear to the totall of the charges, the ouerplus beyng deducted: Wherfore this may you note that before you dooe apply the totall of the charges to the golden rule, you must deduct

e rule.

the

the ouerplus whiche is 6, 12, and 20, that is in the hole 38, but then 8 muſt be reſtored for the abatement of the thirde man, ſo than remaineth to be deducted 30. Take 30 therfore out of 3000, and there will reſte 2970, whiche I muſt ſet in the golden rule for the ſecond ſumme, and for the thyrde ſumme I muſt put eche of the ſmall numbres before mentioned, whiche although they bee not the ſeuerall charges, yet they repreſent them in proportiō. And ſo makyng for euery mās charge a ſeuerall queſtion, the figures will be 4, whiche I marke with 4 letters a. b. c. d, thus:

a

21 ∠ 2970
6 ∠ 848 4/7

b

21 ∠ 2970
4 ∠ 565 5/7

c

21 ∠ 2970
8 ∠ 1131 3/7

d

21 ∠ 2970
3 ∠ 442 2/7

Where I haue ſet for breſenes the ſumme of euery mans charge in the fourth place, preſuppoſyng that you can tell how to try oute that fourthe ſumme, by ſo many examples as ye haue had. Sco. As I truſt that I doo vnderſtande this forme, ſo I deſyre muche to know what may be ſaid for them that miſtoke this queſtion. M. You ſeme ſo deſirous to knowe their errour, that you haue forgotten to examyn whether this worke be without errour. S. Me ſemeth this worke to be

well done, bicause the addition of the iiii. seuerall numbres doth make the total summe of 2970, whiche was to be diuided into suche iiij. partes. M. But then haue you forgotten that the fyrst man must pay 6 crounes more beside this share, and the seconde man 12 crownes more, the thyrde man 8 crownes lesse, and the fourth man 20 crownes more, for without these your fyrst totall of 3000 crownes wyl not be made. S. Then muste I adde to the fyrst mannes summe 6 more, and it wil be 854 $\frac{4}{7}$, and to the secõd summe I must adde 12, & it wyll be 577 $\frac{5}{7}$, from the thirde summe I must abate 8, and then will the summe be 1123 $\frac{3}{7}$, then addyng vnto the iiij. summe 20, it wyll dee 444 $\frac{2}{7}$, and these iiij. summes will make 3000, which is the whole charge, as in this example it may appere, where fyrste I gather the $\frac{14}{7}$, that maketh 2, and so procede I in the addition, to the ende.

	854 $\frac{4}{7}$
	577 $\frac{5}{7}$
	1123 $\frac{3}{7}$
	444 $\frac{2}{7}$
	3000

M. Now haue you well doone, and this woorke in the same summes is brought of other lerned men for the true solution of the question, as it was fyrste proponed, whiche (as I sayd) was impossible, and now examyn it by these seueral summes and se whether it do agree with the summes in the question proponed. The fyrst manne must paye $\frac{1}{7}$, and 6 ouer the totall summe,

howe thynke you is 854 ½ the halfe, and 6 more of 3000? Scho. No that is it not, for it would be 1506, and for the second man 1012, for the thyrde man 1992, and for the fourth man 770, whereof not one summe agreeth to this woorke, but I meruayle that so wyse men coulde bee so muche ouerseene, Maister. It is commonly sene, that when men wyll receaue thynges from elder writers, and wyll not examyne the thynge, they seme rather wyllyng to erre with their ancientes for company, then to be bolde to examyne their workes or writynges, which scrupulositie hath ingendred infinite errors in all kyndes of knowledge, and in all ciuile administration, & in euery kynde of art, but these learned men dyd not meane any other thyng by this question, then to fynde suche numbers as should beare the same proportion togyther, as those numbres in the question proponed, dyd beare one to an other, which thyng you shall perceaue more plainly by an other question of theyrs, that is this. A man lyeng on his deathe bedde bequeatheth his goodes (whiche were woorth 3600 crownes) in this sorte. Bycause his wyfe was great with chylde, and he yet vncertayne whether the chylde were a male or a female, he made his bequest conditionally, that if his wyfe bare a doughter, then shuld the wyfe haue halfe his gooddes, and the

Aqui
of a
ment.

doughter $\frac{1}{3}$, but if she were delyuered of a sonne, thā that son shuld haue $\frac{1}{2}$ of the goodes, and his wife but $\frac{1}{3}$. Now it chanced hir to bryng forth both a sonne and a doughter, the question is, How shal they part the goodes agreably to the testatour his wyll? S. If some cunnyng lawyers had this matter in scannyng, they wolde determyne this testament to be quite voyd, and so the man to die vntestate, bycause the testament was made vnsufficient, sith this condition was not expressed in it, and also it myght haue thaunced that she shuld haue brought forth neither sonne nor doughter, as often hath ben seene, so is the wyl vnsufficient in that point also. Maister. Suche scanners shuld seeme to cunnyng, and yet not so cunnyng as cruell, for the mynde of the testatour is to bee taken fauourably, for the ayde of the legataries, when there ryseth suche doubtes: But lette vs trye this worke not by force of lawe, but by proportion arithmeticall, keeping the testatour dyd mynde to prouide for eche sorte of them. Scholar. If the sonne shall haue $\frac{1}{2}$ by force of the testamente, so must the mother haue $\frac{1}{2}$. Agayn bycause she hath a doughter also, therfore ought she to haue $\frac{1}{2}$, and the doughter $\frac{1}{3}$, that is bothe wayes, $\frac{1}{2}$ $\frac{1}{3}$ and $\frac{1}{2}$ $\frac{1}{3}$, whiche cometh to the hole goodes, and $\frac{2}{3}$ more. Wherfore it seemeth also impossible. Maister. In this matter

matter the mynde of the testatour is so to be vnderstande, that suche proportion shuld bee betweene the portion of the wyfe and the sonne, as is betwene $\frac{1}{3}$ and $\frac{1}{2}$, that is, the sonne must haue $\frac{3}{6}$ for $\frac{2}{6}$ to his mother, so shall he haue 3 to 2, that is as muche as his mother, and halfe as muche more, and the mother must haue the lyke rate in comparison to her doughter. Then must I synd out 3 noumbres in suche proportion, that the fyrst maye bee as muche as the seconde, and halfe as muche more (that is in proportion sesquialtera) and the seconde to the thyrde in the same proportion, suche numbres be 9 6 4. Scholar. I pray you syr, how shall I synde out those numbres? Maister. That will I gladly tell you, what soeuer the proportion be for any 3 numbres. Multiply the termes of that proportion togither, and the numbre that amounteth shalbe the myddle noumbre of the 3, theune multiply that myddle numbre by the leiser terme, & diuide that totall by the greater, and the least numbre of the 3 will amount. So if you multiply that myddle numbre by the greater extreme, and diuide that totall by the leiser extreme, then wyll the greatest noumbre of that progression amounte. Scholar. Then in this example, to synde the proportion of $\frac{1}{2}$ to $\frac{1}{3}$, I muste diuide (as you taughte me in diuysion) $\frac{2}{6}$ $\frac{1}{6}$ by $\frac{1}{6}$, and the quotient wyll be $\frac{2}{3}$,

To fynd numbres any proportion.

that is 1 ½. wherby I perceaue that the pro portion in this question is, as 3 to 2. Ther fore (as you taught me euen nowe) I multiplye 3 by 2, and the summe is 6, whyche must be the myddle numbre, then I multiply that myddle numbre 6 by 2, whyche is the least tearme, and the summe is 12, that doo I diuide by 3, beyng the greater terme, and the quotient is 4, so is 4 the least numbre of the 3, then I multiply 6 by 3, wherof cometh 18, and that I diuide by 2, and so haue I 9, whyche is the greatest numbre of the 3. Mai. An other waye yet may you fynde the thyrd numbre in any progression, yf you haue 2 of theym, for yf the myddle numbre be one of theym, whiche you haue, then multiply it by it selfe (as in this example, 6 by 6 maketh 36) and that totall diuide by the other numbre whiche you haue, and the thyrde numbre wyll be the quotient. Scho. Then if I diuyde 36 (whych cometh of 6 multiplied by it selfe) by 4, the quotient wyll be 9, and yf I diuide 36 by 9, the quotient wyll be 4. But what yf I knowe the fyrst numbre and the thyrd, and woulde haue the myddle numbre? M. Multiply the 2 numbres togither, and in theyr totall you must seeke the roote of that numbre, and it shall be the myddle numbre: but because as yet you haue not learned how to extract rootes, therfore vse the fyrst forme whyche I

haue

haue taught you, tyll I teach you to extract rootes. And now go forward with the answere to the question. Sco. I perceaue then that the sonne muste not haue $\frac{1}{2}$ of the goodes, nother the mother $\frac{1}{3}$, nor yet the doughter $\frac{1}{4}$, but yet must the goodes bee diuided in such proportion, that the sonne shall haue 9 crownes, for 6 to his mother, and the mother shall haue 6 crounes for euery 4 to the doughter. Then I applye it to the golden rule in 3 exaumples thus.

$19 \; 9 \; Z \; 3600$

where the fyrst numbre is the addition of those 3 numbres, 9 6 4, and the third is one of them seuerally, the seconde is the totall of the goodes in the testament, and then by the worke of the golden rule, I fynde oute the fourthe numbre in euery worke, that is for the sonne $1705\frac{5}{19}$, for the mother $1136\frac{16}{19}$, and for the doughter $757.\frac{17}{19}$. whiche three summes added togyther doo make the summe of the whole goodes, as may be sene by this exãple folowyng.

$19 \; 6 \; Z \; 3600$

$19 \; 4 \; Z \; 3600$

$1705\frac{5}{19}$
$1136\frac{16}{19}$
$757\frac{17}{19}$
$3600.$

And this (me thynketh) I doo perceaue that bycause in this case there is a necessary remedy diuised agaynst an vrgent inconuenience, therefore those learned men thought they myght vse

the lyke libertie in that other queſtion. M. Your geſſe is good, but they had not ſo good reaſon for them in the one, as they haue in the other, as in an other example of theirs, it may better appeere, that is this. A man In other queſtion of teſtamēts left vnto his thre ſonnes 7851 crounes, to be parted in this ſort, that the fyrſt ſon ſhuld haue $\frac{1}{2}$, the ſecond ſonne $\frac{1}{3}$, and the third ſon $\frac{1}{4}$, whiche is not poſſible, for $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ doothe make $\frac{26}{24}$ or $1\frac{3}{12}$, that is $1\frac{1}{12}$, ſo is it more then the hole, but reduce theſe fractions in to one denominatiō, and they will be $\frac{6}{12}$, $\frac{4}{12}$ $\frac{3}{12}$, and ſo may you parte the goodes in ſuche proportion as theſe 3 numeratours beare togyther, that is the fyrſt to haue 6 for euery 4 to the ſecond, and the ſecōd to haue 4 as often as the thyrd hath 3, and ſo their portions will be for the fyrſte 3623 $\frac{7}{13}$, for the ſecond 2415 $\frac{9}{13}$, and for the thyrd 1811 $\frac{10}{13}$ and theſe numerators added togi ther wil make the total ſumme of the hole goodes, as you may eaſyly ſe in this example annexed.

3623	$\frac{7}{13}$
2415	$\frac{9}{13}$
1811	$\frac{10}{13}$
7851	

Another queſtion is there proponed thus. In other like queſtion. There is 450 crounes to bee deuided bee twixte 3 men, ſo that the fyrſte man muſte haue $\frac{1}{2}$ $\frac{1}{3}$, the ſecond man $\frac{1}{3}$ $\frac{1}{4}$, the thirde man ſhall haue $\frac{1}{4}$ $\frac{1}{5}$. Scholar. I meruaile that any manne ſhoulde bee ſo ouerſeene to propounde that queſtion as a thynge poſſi-

ble,

ble, syth $\frac{1}{2}$ $\frac{1}{3}$, $\frac{1}{3}$ $\frac{1}{4}$, $\frac{1}{4}$ $\frac{1}{5}$, dooe make $1\frac{47}{60}$ that is almoste double the whole summe. But I perceaue it myghte bee thus propo-ned, that as often as the fyrste man dyd re-ceaue 50 crounes, so often the seconde man shulde receaue 35, and the third man 27, for $\frac{1}{2}$ $\frac{1}{3}$ is equall to $\frac{50}{100}$, and so is $\frac{1}{3}$ $\frac{1}{4}$ equall to $\frac{35}{100}$, and $\frac{1}{4}$ $\frac{1}{5}$ is $\frac{27}{100}$, and so working the que stion, the 3 sygures wyll appeare in this forme, wherby the fyrste mans portion is found to be 200 $\frac{50}{56}$, the secõd mãs part is 140 $\frac{35}{56}$, the third mannes share is 108 $\frac{27}{56}$, whiche in the whole dothe make 450 crounes, that was the whole summe to be diuided betwene them.

112 — 450
50 — 200 $\frac{50}{56}$

112 — 450
35 — 140 $\frac{35}{56}$

112 — 450
27 — 108 $\frac{27}{56}$

Mai. And thus you are (I thynke) sufficiently instructed in the rule of felowshyp.

The Rule of Alligation.

NOwe wyll I go in hand with the rule of Alligation, whiche hath his name, for that by it there are diuers parcels of sundry prices, and sundry quantities alligate, bounde, or myxed togyther, whereby also it myghte be wel called the rule of Mixture, and it hath great vse in composition of medicines, and also in myxtures of mettalles, and some vse it hath in myxtures of wynes, but

The rule of mixt

I

I wyshe it were lesse vsed therein than it is now a daies. The order of the rule is this, Whē any summes ar proponed to be mixed set them in order one ouer an other, and the common numbre whervnto you wyll reduce them, set on the left hand, then marke what summes be lesser then that cōmon numbre, and whiche be greater, and with a draughte of your pen, euermore linke .ii. numbres togither, so that one be lesser then the cōmon numbre, and the other greater then he, for ij. greater or ij. smaller can not well be lynked togyther, & the reason is this, that one greater and one smaller may bee so mysed, that they wyl make the meane or common numbre very well, but .ii. lesser can neuer make so many as the cōmon numbre, beyng taken orderly, no more can .ii. summes greater thā the meane, neuer make the meane in due order, as it shal appere better to you hereafter. And as it is of necessitie to linke euery smaller ones (at the leaste) with one greater, and euery greater with one smaller, so it is at libertie to lynke theim oftener then ones, and so may ther be to one questiō many solutiōs.

he reson this ake.

When you haue so linked thē, then marke how much eche of the lesser nōbres is smaller then the meane or common numbre, and that difference set agaynst the greater numbres whiche bee lynked with those smaller, eche with his matche stil on the right hand,

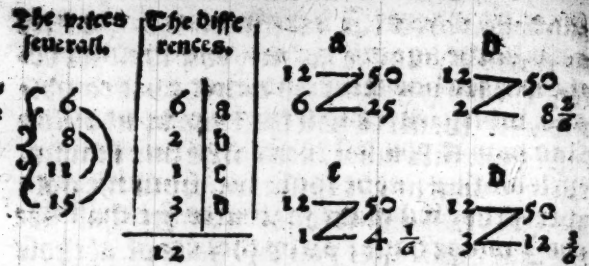
and

and lykewayes the excesse of the greatter numbres aboue the meane, you shall set before the lesser noumbres whythe bee combined with them. Than shall you by addition bryng all these differences into one summe, whyche shall be the fyrst numbre in the golden rule, and the seconde numbre shall bee the hole masse that you wyl haue of al those particulars, the third summe shalbe eche difference by it selfe, and than by theym shall be founde the fourthe numbre, declarynge the iuste portion of enery particular in that mixtion. As now by examples I wyll make it playne.

There is. iiii. sortes of wyne of seuerall prices, one of 6 d a galon, an other of 8 d, the thyrde of 11 d, and the fourth of 15 pens the galon, of all these wynes wolde I haue a myxture made to the summe of 50 gallons, and so that the pryce of eche galon maye be nyne pens. Nowe demaunde I howe muche muste bee taken of euery sorte of wyne? Scholar. If it shall please you to worke the fyrst example, that I may marke the applyeng of it to the rule, then I truste I shall bee able not onely to dooe the lyke, but also to se reson in the order of the worke. Mayster. Marke then this forme, and the placynge of euerye kynde of noumbre in it.

A question of myxing of wyne.

The



Here (you see) I haue set down the seuerall prices which be 6, 8, 11, 15, and haue linked togither 6 with 15, and 8 with 11, the comō price 9 I haue set on the left side, & the difference betwene it and euery particuler price, I haue set on the ryght hande, not agaynst the summe whose difference it is, but against the summe that it is lynked withall, so the difference of 15 aboue 9 is 6, which I haue set not against 15, but against 6, that is linked with 15, and the difference betweene 6 & 9 (that is 3) I haue set agaynst 15, So lyke waies the differēce betwene 8 and 9 is but 1, that haue I set agaynst 11, and the differēce of 11 aboue 9 (whiche is 2) I haue set agaynst 8. Then adde I all those 4 differences, and they make 12, whyche I set for the fyrst numbre in the golden rule, the seconde numbre I make 50, which is the summe of the galons that I wold haue, and the thirde summe is euery particular difference. Nowe if you woorke by the golden rule, you shall fynde the numbre of galons that shall be taken of eche sorte of wyne, for the better di-

stinction wherof, I haue set these letters, a, b, c, d, bothe against the numbres for whiche the worckes doo serue, and ouer the worckes also, which seuerally serue for eche of them. And now if you lyst to examyn the truthe of these worckes, adde those iiii. summes togither, & they wil make 50, that is the total whiche I wolde haue, as by this example you may easyly perceaue. And for to proue howe the prices do agree, do this. Multiply this totall summe 50 by the common price 9, and it wyll make 450, than kepe that summe by it selfe, & afterwarde multiply euery seuerall sum of galons, by the price belongyng to the same galons, & if that sum do agre with this, whi che you haue kept fyrst, then is your worck well done. As here 25 is the numbre of galons of 6 d price, multiply then 25 by 6, and it maketh 150, which you shall set down, then multiply 8 2/6 by 8, which is the price for the nũbre of galons, and it wyl make 66 4/6, so agayn 4 1/6, multiplied by 11, doth make 45 5/6. And last of al 12 3/6 multiplied by 15 maketh 187 3/6. All these added togither dooth make 450, as in the exãple annexed you may se, wherfor seig it doth agre wt the former sum of 50 multiplied by 9, I may iustly affyrme this

The proof of this rule.

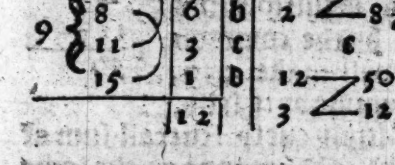
25 — 8 2/6 — 4 1/6 — 12 3/6 — 50

150
66 4/6
45 5/6
187 3/6
450

this woorke to be good and well doone.

The varia-tion of this que-stion.

And nowe to proue howe you can doo the lyke, I propounde the same question, onely applying you to vse some other forme of combining or lynking the summes. Sco. That shal I proue with your fauour, and therfore I combine 8 with 15, and 6 with 11, and then the forme wyl be thus,



whereby amounteth the same summe in totall of the differences, as dyd before, and yet nowe the differences be altered, as the combination is chaunged, whereof I vnderstand the reason by your former woorke. And therfore here appereth no straunge thynge, but that nowe I must haue 8 2/6 galons of 6 d, and 25 galons of 8 d, and 12 galons, and 3/6 of 11 d, and so consequently 4 galons and 1/6 of 15 d. so that multiplying 8 2/6 by 6, it maketh 50, and then 25 multiplied by 8, maketh 200, likewaies 12 3/6 multiplyed by 11, yeldeth 137 3/6, and 4 1/6 multiplied by 15, maketh 62 2/6, whych the iii. summes added in one, wyll yeld

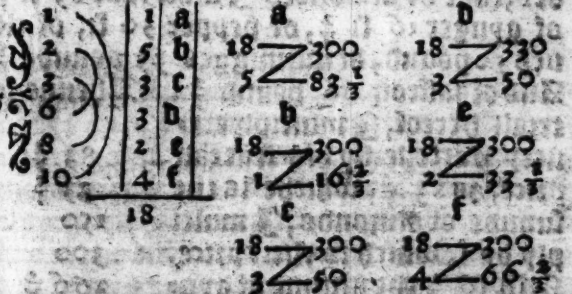
50
200
137 3/6
62 3/6
450

in the

in the totall 450, whiche agreeth with the multiplication of 50 (being the total summe of galons) by 9 the common or meau price.

M. Seyng you conceiue this worke so wel, I will propound an other example vnto you of more varietie in the alligations or combinynges: As thus, A marchaunt beynge mynded to make a bargayne for spices in a myxt masse, that is to saye, of cloues, nutmigges, saffron, pepper, ginger, and almondes; the cloues beeyng at 6 ℔ a pounde, the nutmigges at 8 ℔, saffron at 10.s. pepper at 3.s. gynger at 2.s. and almondes at 1.s. Nowe wold he haue of eche sort some, to the value of 300 ℔ in the whole, and ech pound one with an other to beare in price 5 s. how muche shall he haue of eche sorte? Scholar. That will I trie thus. Fyrst I sette downe those 6 seuerall prices, and at the left hande

X questi of spyce

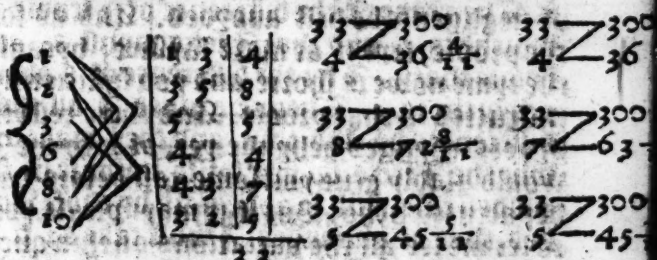


I set the common price 5.s. Then I ynke them thus, 1 with 10, 2 with 6, 3 with 8.

Mai. I had minded to haue combined them in more varietie, but I am contents to set your owne worke first, and then more varieties in combination may folow anone. Sco. Then to continue, as I beganne, I seke the difference betwene 1 and 5 (which is 4) and that I set agaynst 10: then agaynst 1: I set 5, whiche is the excesse of 10 aboue 5, so I gather the difference betwene 2 and 5, whiche is 3, and that I set agaynst 6, bycause it is combined with 2: and likewaies the difference of 6 aboue 5 (which is 1) I set against 2. Then take I the difference of 3 frome 5, whiche is 2, and that I set agaynst 8, and before that 3. I set the difference of 8 aboue 5, whiche is 3. Thenne gather I all these differences by Addition, and they make 18, whyche I sette for my first numbre in the golden rule, and so appeareth by those wordes, that of almondes I must take 83 li $\frac{1}{3}$, of gynger 16 li $\frac{2}{3}$, of pepper 50 li, of cloues 50 pound, of nutmigges 33 pounde $\frac{1}{3}$. And of saffron 66 pound $\frac{2}{3}$. Thenne for triall hereof, I multiplye euery parcelle by his feuerall price, as 83 $\frac{1}{3}$, whiche is the summe of almonds, I multiply by 1, which is their price. Also 16 $\frac{2}{3}$ the summe of gynger, I multiplye by 2, which is the price of it. And so eche other in his kynde, as this

83	$\frac{1}{3}$
33	$\frac{1}{3}$
150	
300	
266	$\frac{2}{3}$
666	$\frac{2}{3}$
1500	

table annexed dooche represente, and then addyng theim all togyther, I fynde the totall to be 1500, whiche also wyll amount by the multiplication of the grosse masse of 300 by the common price 5, wherfore it appeareth well wrought. Mayster. Nawe wyll I make the alligation to proue your cunnyng some what better, but bicause you shall not thynke your selfe prested to muche, I wyll also note the differences, as in this example folowyng you may see, where I haue alligate 1 with 6 and 8, and therefore haue I

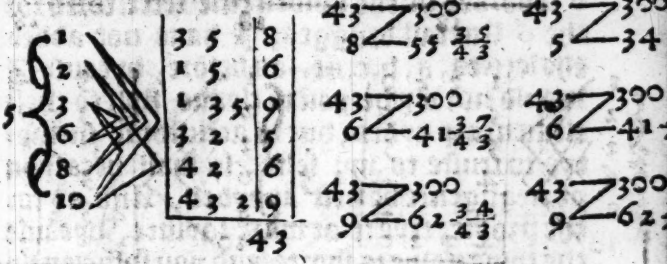


sette against 1, both theyr differences, that is 1 and 3. Likeways bicause 2 is combined with 8 & 10, I set before him their differences 3 & 5. Against 3 I haue set only 5, whiche is the difference of 10, with whom 3 is cõbined only: likewais 6 is only alligate to 1, & therfore is the difference of 1 only set against it: 8 is ioyned with 1 and 2, & therfore hath he against him there both differences 4 and 3: 10 is ioyned with 2 and 3, and therfore hath

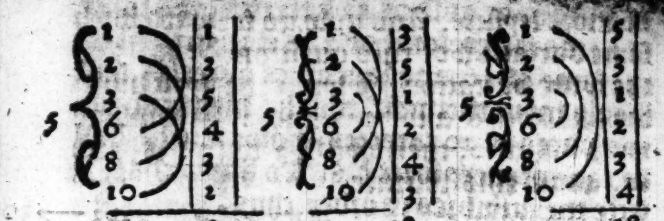
he their differences 3 and 2. And bicause of eale for you, in an other columne, I haue set the differences reduced into one numbre, for euery seuerall sorte, and haue also added them togither, whereby appereth that they make 33. And so consequentely you see the 6 woorkes of the golden rule sette forth for the 6 seuerall dsrugges, I haue not added the letters, a, b, c, &c. as before, bycause I would not wythe you to cleaue styll to these elementary aydes, but to accustome memorye to trust to hyr selfe, so shall occasyon of negligence be best auoyded. And as for the proofe, trye it at more layssure, bycause the tyme nowe is shorte, and you sufficiently instructed in that proofe. And there resteth dyuers thynges behynde yet, of wyche I would gladly geue you some tast before our departure. Scho. But if it may please you to let me see all the variations of this question, before you go from it, for me thinketh I could varye it 15. or 16. wales more yet. Maister. I am content to se you make two or thre variations, but I would bee lothe to staye to see all the variations, for it maye be varyed aboue 300 wales, although many of theim wold not well serue to this purpose. Scho. I thought it impossible to make so many variations. Maist. Mervaile not therat, for some questions of this rule may be varied aboue 1000 wales: but I would haue you forget suche fantasies, tyll a tyme

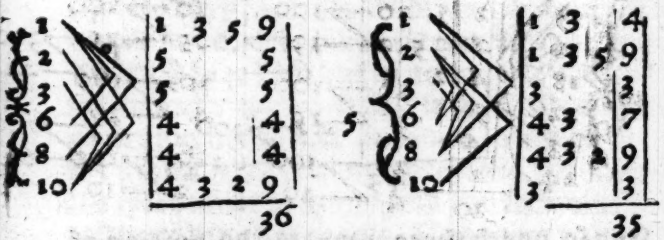
of

of more leysure. And now go forward with some variation of this question. Scho. For the fyrste variation, I lynke the fyrst numbre one with 8 and 10, and 2 I combine with 6 and 10, then ioyn I 3 with 6, 8, and 10, as in this fourme.



And so dooth there appeare the portion of weighte for euery kynde of drugge in this myxture. Nowe for the triall. Mai. Naye staye there, you shall not nede to make triall in one exaumple so often, or if you lyste to doue it by your selfe, I am contente. But now let ooorth (for declaration that you conceaue the rule) two or three examples of seuerall combinations, & then wyll we passe to some other example, and so ende this rule. Scho. As it pleaseth you, so wyll I do. And these be the varieties in whiche as the combinations are seuerall, so dooth it playnely appeare, that the differences, by which the proportion of eche seuerall kynde is taken, ar also seueral, And yet I se in the iij. fyrst



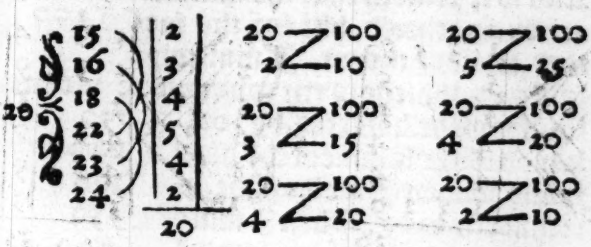


of these. v. varieties, and in one other before. the totall summe of the differences to be one, that is to say 18, whereby I perceaue that the varietie of their mixture dothe depend of the varietie of their differences seuerall, and not of the varietie of their totall sum. M. So is it. And seing you conceaue it so well, I will make an ende of this rule, only exhibityng to you one question or .ij. of the mixture of metals, that by it you maye deuise other like, and exercise your self therin also, bycause the vse of it serueth often in

question mixtynge golde. busynes of charge, not so muche for goldesmythes, as for coynage in myntes. First I demaunde of you this question. If a mynt

mayster

mayster haue gold of 22 karectes, and some of 23 karectes, some of 24. Agayn some of 15, some of 16, and som of 18 karectes, and would myxe theim so, that he myghte haue 100 vnces of 20 carectes, howe muche shall he take of euery sorte? S. To know that, I sette the numbres in order thus.



M. You haue wrought the question well, but howe chaunced, you made no doubte of that newe name Karect. Scho. Bicause I thoughte it out of tyme to demande suche questions nowe, seyng you make so muche haste to end, and agayn in this case the proporcion of the numbres is sufficient for my pourpose in this woorke, trustyng that an other tyme you will instructe me as well of this as of sundry other thynges, whych as I haue hearde you to talke of, so I haue a greate desyre to knowe theym. M. Your answere is reasonable, and your request and trust, with gods helpe, I intend to satisfye.

And nowe to go forwarde with this matter, lette me see youre examination of this laste worke. Sco. First for the one parte, I adde togyther all the particular summes, as they appeare in the worke, and they make 100, as here by their addition it dooth appeere. And so it semeth that the summes are well gathered, but for the farther triall of theym, I multiplye first 20, whiche is the common or meane summe of the karectes, by 100, which is the summe of the hole masse, whiche I wolde haue, & it maketh 2000. Then I multiply euery particular summe by the karectes that it doth cõteyn, as 10 by 15, and that maketh 150. Lykewais I multiply 15 by 16, and it yeldeth 240, so 20 by 18 maketh 360. And 25 by 22 yeldeth 550, like waies 20 by 23 bryngeth foorthe 460, and laste of all 10 multiplied by 24, yeldeth 240 whyche summes all ioyned togyther make 2000, that dooth agree with the like summe before, wherfore I may well saye, that the worke is good. And nowe if it please you, I woulde sette foorthe somme varieties of this question, to proue my wytte. Maister. Do so, lette me see. Scholar. Here be foure varieties.

10	
15	
20	
25	
10	
100	
150	
240	
360	
550	
460	
240	
2000	

And



And moze yet I could make, but not lyke to the noumbze that you ſpake of in the variation of the other queſtion. Mai. That wylle I teache you at moze leiſer, ſeyng it is a thyng rather of pleaſure, then of anye neceſſitie.

But now foz your exercitſe in this rule one other queſtion I wyll pzopoſe. A mynt maſter hath 6 ingottes of ſiluer, of ſundzy fineneſſe, ſome of 4 vnces fyne, and ſomme of 5 vnces, ſome of 6, and other of 8, ſome of 11, and other of 12, And his deſyze is to mixe 500 pound weyght, ſo that in the hole maſſe, euery pound waight ſhoulde beare 9 vnces

A queſt of mixt of ſyl

vnces of fyne syluer, howe muche shall he take (say you) of euery sorte of syluer? S. Iyo. To fynde out that, I set the numbres thus in order.

4	2		2
5	2		2
6	3		3
8	3		3
11	5	4	9
12	3	1	4
			23

And gatheryng the differences, it wyll apere, that of the fyrste sorte there muste bee 43 $\frac{1}{2\,3}$, of the second lyke muche, of the thyrde sorte, 65 $\frac{5}{2\,3}$, and of the fourthe sorte as muche, of the .v. sort 195 $\frac{1\,5}{2\,3}$, and of the syxt sorte 86 $\frac{2\,2}{2\,3}$, whiche in the whole wyll make 500 li weyght, and in vnces 4500, that is of the fyrst sorte 173 $\frac{2\,1}{2\,3}$, and of the secõd sort 217 $\frac{0}{2\,3}$, of the third sort 391 $\frac{7}{2\,3}$, of the .iiij. sort 521 $\frac{1\,7}{2\,3}$, of the v. sort 2152 $\frac{4}{2\,3}$, and of the vi. sort 1043 $\frac{1\,1}{2\,3}$, whiche all togyther doo make 4500 vnces agreeable to the multiplication of 9 by 500. M. This is well done of you, ther sore nowe make iii. or. iiii. varieties, and so an end of this rule. S. Yeele four varieties I set for example.

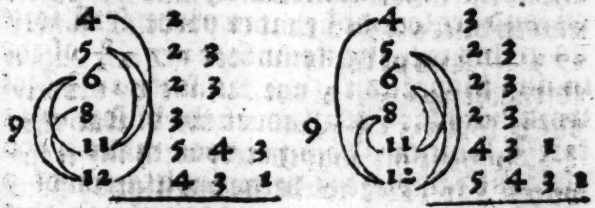
4	3		3
5	3		3
6	3		3
8	2		2
11	1		1
12	5	4 3	12
			24

9

4	2	3	5
5	2		2
6	2		2
8	2		2
11	5	4 3 1	13
12	5		5
			29



And by these it appereth that you can fynde out more, with which I wyl not now medle, saue onely for to shewe you an easy helpe in drawynge the lynes of combination, I will sette foorthe two varieties here.



And this shall suffise nowe for the rule of alligation or mixture, for by these exaumples may you easily coniecture suche other as do appertayn to it, as wel for the due worknyng as for variete of drawyng the lines of combination.

The Rule of falsehode.

NOwe wyll I breefely also teache you somewhat of the rule of Falsehoode, which

whiche beareth his name, not for that it teacheth any fraude or falsehoode, but for that by false numbres taken at all aduentures, it teacheth howe to fynde those true numbres that you seke for. Sco. So myght any other rule be called the rule of falsehode, for they worke by wrong numbres, and by them find out the ryght numbres, so dooth the rule of Alligation, the rule of fellowshyp, and the golden rule partly. Ma. In the golden rule, the rule of felowshyp, and the rule of Alligation, although the numbres that you worke by, be not the true numbres that you seeke for, yet are they numbres in iuste proportion, and are founde by orderly worke, where as in this rule the noumbres are not taken in any proportion, nor founde by orderly work, but taken at all aduentures. And therfore I, somtimes being merye with my frendes, and talkynge of suche questions, haue caused them that proponed suche questions to call vnto them suche children or ydeotes, as happened to bee in the place, and to take theyr answere, declaryng that I wold make them to solue those questions that semed so doubtfull. And in dede I dyd answer to the question, and worke the tryall therof also, by those answeres whiche they happened at all aduentures to make, whyche numbres seyng they be taken as manyfest false, therfore is this rule called the rule of false posi-

tions

tions, and for breuenes. The rule of falsehod whiche rule for redynes of remembrance, I haue cõprised in these few verses folowyng in forme of an obscure ryddle:

G *From manny bate to many mo,*
H *From to fewe, take to fewe also,*
I *With to muche ioyne to fewe agayn,*
K *To to fewe adde to many playne.*
L *In crosse waies multiply contrary kynde.*
M *All truthe by falsehode for to fynde.*
C *And fyrst woorke by the question,*
D *Although no truthe therin be done.*
E *Suche falsehode is so good a grounde,*
F *That truthe by it wyll soone be founde.*
A *Gesse at this worke as happe doth leade.*
B *By chaunce to truthe you may procede.*

The sence of these verses, and the summe of this rule is this, When any question is propoued appertaynyng to this rule, first imagyn any numbre that you lyste, whiche you shall name the fyrst position, & put it in stede of the true numbre, and then woorke with it, as the question importeth, and yf you haue mysled, then is the last nũbre of that worke other to great, or to lytle, that shall you note, as hereafter shall bee taughte you, and you shall call it the fyrste errour. Then begynne agayne and take an other noumbre, whiche shall be called the seconde position, and woorke by the question, yf you haue mysled,

agayn

agayne, note the exceſſe or defaulte as it is, and call that the ſeconde errour. Thenne multiplye croſſe wayes, the fyrſte poſition by the ſeconde errour, and agayne the ſeconde poſition by the fyrſte errour, and note theyr totalles ſeuerally by the names of totalles. Then marke whether the two errours were bothe lyke, that is to ſay, bothe to muche, or bothe to lytle, or whether they bee vnlyke, that is, the one to muche, and the other to lytell. for if they bee lyke, then ſhall you ſubtracte the one totall frome the other (I meane the leſſer from the greater) and the remayner ſhall bee your diuidende, ſo muſte you abate the leſſer errour oute of the greater, and the reſydue ſhall be the diuiſour. Nowe diuide the diuidende by that diuiſour, and the quotient wyll ſhewe you the trewe numbre that you leke for. But and if the errours bee vnlyke, then muſte you adde bothe thoſe totalles (whych you noted) togither, and take that hole numbre for the diuidende; ſo ſhall you adde bothe the errours togyther, and that hole numbre ſhall be the diuiſour. And the quotient of that diuiſion ſhall geue you the trewe numbre that the queſtion ſeketh for. And this is the hole rule. Scholar. This rule ſeemeth ſo vnlyke any other, that without ſom example I ſhal not eaſyly vnderſtande it. M. Wherfore take this example: A maſon was bound to build

a walle

a wall in 40 daies, and it was couenanted so with hym, that euery day that he wrought he shoulde haue for his wages 2 ß 1 ð, and euery day that he wrought not, he shoulde bee amerced 2 shyllynges fyue pens, so that when the wall was made, and the reckonyng taken of the dayes that he wrought, and of the other that he wrought not, the mason had clearely but 5 s. 5 ð for his worke. Now do I demand how many dais dyd he work of those 40, and how many did he not worke? Scho. I praye you expresse the order of the worke, that I maye partely by imitation, and partly by comparyng it with the rule, bee hable agayne to doo the lyke. M. This order shall you kepe in the worke of this rule, fyrst take some numbre as you lyst at aduenture, as for example, I say, he plaied 12 daies and wrought 28 daies. Now caste you the wages of euery daye, and see whether it wil agre with the summe of 5 s, 5 ð. Scho. The 28 daies that he wrought, after 25 ð by the day, yeldeth 700 ð. Then the 12 daies that he wrought not, at 30 pens ethe daye dothe amount to 360 pens, whiche yf I abate out of 700, ther resteth 340: but you say he had not so muche. Maist. He had but 65 pens, and by this supposition he shuld haue had 340, therfore is this sum to much by 275, whiche summe I muste serue doune after this sort, as you see here, where fyrste

A queste of mason

Fyrste I haue made a
croſſe (commonly caled
ſainct Andrewes croſſe)
at the ouer corner on
the lefte hande I haue



ſet the fyrſt poſition 12, and at the other cor
ner vnder it, I haue ſet 275, whiche is the
fyrſt errour, with this figure +, whiche be
tokeneth to much, as this lyne, —— plain
without a croſſe line, betokeneth to litle. On
the ryght hande of the croſſe I haue left ſi.
lyke roumes for the ſecond poſition and his
errour. Therfore to proſecute the worke I
ſuppoſe he playde 16 dates, and wrought
24. Scho. I was a whyle in doubte why
you named the dates of his workyng, ſeyng
they be not ſet in the figure, and I doubted
howe you knewe theim, or els whether you
dyd ſuppoſe them at all aduentures, as you
dyd the dates that he played, but now I ga
ther, that ſeyng 40 dates is the hole time li
mitted, then the days that he played beyng
ſuppoſed, the reſte of 40 muſt neves be the
dates that he wrought, and therfore 28 folo
wed 12 of neceſſitie, and 24 followeth 16
alſo of neceſſitie: but yet I ſcarſe perceaue
why you ſet not in the figure as well 28 as
12. M. It forceth not, whiche of theym I
take, ſo that in the ſeconde poſition I take
the nombre of the ſame nature, that is bete
both of workyng dais, or bothe of yole dais,

but

the old denominator, it wold haue ben not only alwel done, but also into a fraction of lesser termes. S. I gesse it to be euē so, by a like worke that you taught me in Multiplication. And for proofe therof $\frac{20}{23}$ beyng the diuidend, and 4 the diuisor, I diuide the numerator 20 by 4, & the quotient is 5, whiche I set for 20 ouer 23, thus $\frac{5}{23}$. And I see that it is all one with $\frac{20}{92}$, as by dyuydyng both these termes by 4, and so reducyng them to their least denominatiō, I may easily proue. M. You conceue it wel. And if there be mixt numbers (other one or both) you must firste reduce that mixte numbre into an improper fraction. And then worke as you haue lerned S. That was sufficiently taught in Multiplication. Therfore I pray you go forward to some other thing. M. Then take this note yet for diuision. If the denominators be like then diuide the numerators, as if it were in hole numbres, and the quotient, whether it be fraction, hole numbre, or mixt, is a good quotient for that diuision. And generally, if one of the numerators may iustly diuide the other, by that quotient multiply the denominator of the lesser numerator, and set it that doth amount in the rome of the same denominator, and then for a numerator to it, set the denominator of the other fraction. Stol. Then yf I woulde dyuide $\frac{3}{4}$ by $\frac{1}{12}$, I see that 3 wil diuide 12, and the quotient wil be

4 that is denominatour vnder 3, and then is it 16, whiche I sette for the denominator 4, and ouer it in steede of the 3, I muste sette 17, the other denominatour, and so is it thus $\frac{17}{16}$. Mai. And so is $\frac{17}{16}$ in steede of $\frac{51}{48}$ whyche woulde haue ryſen by the com-
ediation mon wooꝛke.

And nowe foꝛ Mediation (which is to diuide by 2) mark this: If the numeratour be euen, set the halfe of it in his place, without the diuisour, and so haue you doone. And if the numeratour be not euen then double the denominatoure. Scholar. That is, yf I woulde mediate $\frac{6}{11}$, I may make the quotient $\frac{3}{11}$. And yf I wold mediate $\frac{7}{11}$ I muste make it $\frac{7}{22}$. Maister. Nowe truste I that you haue sufficiente knoweledge in Reduction, Multiplication, and Diuision: And therfoꝛe wyll I go in hande with Addition and Subtraction, whiche nowe wyll appere easye ynough.

ADDITION.

When so euer you haue any fractions to be added, you muste consyder whether they be of one denomination oꝛ not. And if they be of one denomination, then adde the numeratours togyther, and set that amounteth foꝛ the numeratoꝛ ouer the cõmon denominatoꝛ, and so haue you doon. But and yf the fractions be not of one denominatiõ

oꝛ

or any of them, be myxte of whole numbres and fractions, then muste you fyrste reduce them to one denomination, and after adde them. And if they be many, then adde firste two of theim, and to the summe that bothe amount of that addition, adde the thyrde, and then the fourthe, and so forth, yf you haue so many. Scho. This seemeth easy ynough, now that I haue all ready learned to multiply, and to reduce, without whiche two, I could neuer haue wrought this. And therfore nowe I see good reason, why you bydde place Multiplication and Reduction before Addition. Maister. It is well consydered, but yet refuse not to exprisse youre vnderstandyng of it by an example. Schol. Then would I adde fyrst $\frac{7}{1\,8}$ with $\frac{5}{1\,8}$, and bycause the denominatours are lyke (and so needeth no Reduction) I adde 7 to 5, whyche maketh 12, and then is my summe $\frac{12}{18}$, that is in smaller numbres $\frac{2}{3}$. And yf I haue many numbres to be added, as here $\frac{3}{8}\ \frac{4}{5}\ \frac{9}{10}$, fyrst I must reduce them (bicause they haue dyuers denominatours) into one denomination, and thenne will they bee thus. $\frac{15}{40}\ \frac{32}{40}\ \frac{36}{40}$, or in lesser termes $\frac{15}{40}\ \frac{32}{40}\ \frac{36}{40}$, which by addition do make $\frac{83}{40}$ that is $2\frac{3}{40}$. Mai. Nowe may we go to subtraction.

Subtraction hath the same preceptes that Addition had, for if the denominatours be lyke, then must you subtract the one numeratour from the other, and the rest is to be set ouer the common denominator, and so is your subtraction ended: but and if you haue many fractions to bee subtracted out of many, then must you reduce them to one denomination, and into two seuerall fractions, that is, all that must bee subtracted into one fraction, and the residue into an other fraction, and then worke as I said before. Scholar. For the fyrste exaumple I take $\frac{5}{12}$ to be subtracted out of $\frac{7}{12}$, and the reste wyll be $\frac{2}{12}$ or $\frac{1}{6}$: for an other exam ple I take $\frac{3}{4}$ to bee subtrahed oute of $\frac{7}{8}$, wherefore I must fyrst reduce, and it wyll be thus. $\frac{24}{32}$ and $\frac{28}{32}$, then doo I subtract 24 oute of 28, and there resteth 4, which I set ouer the common denominator for a remainer, thus $\frac{4}{32}$ that is $\frac{1}{8}$. Nowe for the thyrd example I take $\frac{3}{4}$ & $\frac{5}{6}$ to be subtrahed from $\frac{7}{8}$ and $\frac{9}{10}$ And because their denominatours be dyuers, I doo reduce them thus, $\frac{1440}{1920}$, $\frac{1600}{1920}$, $\frac{1680}{1920}$ $\frac{1728}{1920}$. Then doo I adde the twoo fyrst, and they make $\frac{3040}{1920}$. Also I adde the two laste, and they yelde $\frac{3408}{1920}$, then I do subtracte 3040, out of 3408, and there resteth 368, so is the remainer $\frac{368}{1920}$, that is in smaller termes $\frac{23}{120}$. And thus haue I doone

done with Subtraction except you haue any more to teach me. M. Proue one example more of two fractions of diuers denominations. Sco. I take these two fractions, $\frac{7}{8}$ and $\frac{9}{24}$, whyche beynge reduced wyll stande thus. $\frac{168}{192}$ and $\frac{72}{192}$. Now would I subtract 168 out of 72, but I can not. M. Then may you perceue that you mistoke the fractions, for you can neuer subtract the greater out of the lesser, although you may adde, multiply, or deuide the greater with the lesser. And all be it that $\frac{7}{8}$ hath both hys rearmes lesser then $\frac{9}{24}$, yet is $\frac{9}{24}$ the lesser fraction, for generally yf you multiplye the numeratours and denominatours of two fractions crosselwayes, that fraction is the greatest of whose numeratour cometh the greatest sum as in thys example 7, multiplyed by 24, maketh 168, and 9 beyng multiplyed by 8, yelldeth but 72, therfore is the fyrst fraction $\frac{7}{8}$ the greatest of these two, so can you not subtract it out of a lesser fraction. But and you shuld subtract a fractiō out of a whole numbre, what wold you do? S. Marry I wold reduce that hole numbre into a fraction of the same denominatiō, that my fraction is, and then wourke by Subtraction. Mayster. So maye you doo, but it is ealyer much yf youre fraction bee a propre fraction, that is to saye, lesse thenne an vnyte, to take an vnyte from the whole noumbre, and

The greatest of two fractions

and thenne tourne it into an improper fraction, and so worke your Subtraction. As yf I woulde subtracte $\frac{3}{4}$ frome 4, I maye take one frome 4, and tourne it into $\frac{4}{4}$, frome whyche yf I abate $\frac{3}{4}$, there wyll remayne $3\frac{3}{4}$. And if the fyrste fraction be an improper fraction, then maye I take so many vnitees frome the whole numbre, that they may make an improper fraction greater then that fyrst, and then worke the subtraction: As yf there bee proponed $\frac{1}{3}$ to subtrahed from 6, bicause $\frac{1}{3}$ is more then 3, and not 4, I muste take 4 frome 6, and tourne theym into thyrdes thus, $\frac{12}{3}$, than abate $\frac{10}{3}$, and there resteth $\frac{2}{3}$, so the whole remayner is $2\frac{2}{3}$. And thus wyl I make an ende of the works of common fractions for this tyme, not doubtyng, but you can apply them bothe vnto the rules of Progression, and also vnto the golden rule, without any other techyng then you haue learned before, which myght seem tedious to repete, saue yt in some spectall diuersities, whyche be peculiar to fractions, I can not ouerpasse, but instruct you somwhat by the way.

The Golden Rule.

Therfore as touchyng the golden Rule for the placyng of the iij. numbres proponed in the question, wherby to fynd the thyrd, and for the tryme of their worke, with other lyke notes, I referre you to that whyche you haue already lerned. But

But thys easye fourme of wooꝛkynge by fractions shall you note, that yf your thꝛee noumbꝛes be fractions foꝛ an aple woꝛke & certayne, multiplye the numeratoure of the fyꝛste noumbꝛe in the question, by the denomynatour of the second, and all that againe multiplye by the denominatour of the thyꝛde noumbꝛe. And the totall thereof shall you keepe foꝛ to bee the diuisoꝛ: Then multiplye the denominatoure of the fyꝛste numbꝛe by the numeratoure of the seconde, and the whole thereof by the numeratoure of the thyꝛde, and the totalle thereof shall bee your diuidend. Nowe dyuide thys dyuydend by the dyuisoure whyche you founde out before. and that numbꝛe shall be the iuſt noumbꝛe of the question, whyche you seeke foꝛ. As in thys example, If ¾ of a yarde of velues coste ⅔ of a souerayne, (esteemed at 20 shyllynges) what shall ⅚ coste? Scholar. If it please you to lette me make the aunswer, I would fyrst place these thꝛee numbꝛes, as I learned in whole numbꝛes, thus ¾ ⅔ ⅚ And then accoꝛdyng to youre new rule, I must multyply 3 beyng numeratour in the fyꝛst noumbꝛe, by thꝛe the denominatour of the second, and therof cometh 9. whiche I multiply agayne by 6, the denominatour of the thyꝛd numbꝛe, and so haue I 54, whiche I keepe foꝛ the diuisour, then multiply I 4 the denominatoꝛ of the fyꝛste, by 2.

A queſt of valu

the numeratour of the second, and there ryseth 8, whych agayne I multyplye by 5, the numeratour of the thyrd, and it maketh 40; then must I dyuyde 40 by 14, and it wyl be $\frac{40}{14}$ that is $2\frac{12}{14}$, in lesser termes, and then the figure wyll stande thus. $\frac{1}{4} \angle \frac{2}{7}$ $\frac{1}{6} \angle \frac{2}{14}$ But what that is in mo- nye, I can not tel, except I shal woorke it by reduction, as you taught me. Mai. It forteth not nowe, you maye reduce it when you lyst, but it were disorderly done here to mingle dyuers woorkes togyther, where we doo not seeke the valew of the thyng in common mony, but in an apte numbre, whyche you haue well done. And therfore will I yet shew you an other lyke way of easynes in woorke, howe you maye chaunge youre three fractions into 3 whole numbres, by whych you shall woorke as yf the question were proponed in whole numbres. The fyrste numbre you shall fynd, as I taught you now to find the dyuisour. for the second numbre take the numeratour of the second fraction, and for the thyrde numbre take that, that ryseth of the multiplication of the denominatour of the fyrste, by the numeratour of the thyrde, and then woorke youre question. S. For exam- ple hereof, I putte thys question, If $\frac{1}{12}$ of 1 li. waight of syluer be worth $\frac{1}{4}$ of a soue- raygn, what is $\frac{1}{2}$ of 1 li. weight woorth? For the aunswere, fyrste I place the fractions in

Question of syluer.

order

order thus. $\frac{1}{1}\frac{1}{2} \angle \frac{2}{4}$
Then to tourne these fractions $\frac{1}{2}$ into whole numbres, I multiply 11 which is the numeratour of the fyrste, by 4 (the denominatour of the second) and there cometh 44, whych I multiplye by 2, the denominatour of the thyrd, and so amounteth 88, whiche I sett for the Denisour in the fyrst place. Then in the second place I sett 12, which is numeratour in the second fractiõ, and in the thyrd place I sette the summe that amounteth of 12, beeyng the denominatoure in the fyrste nombre multiplyed by 1, beyng numerator in the thyrd numbre, and so the figures wyll stãd as here you see. Thẽ 88 — 12 to worke it forth, I multiply 12 — 12 by 12, and there amounteth 144, whych I dyuyde by 88, and the quotiente wyll bee $1\frac{56}{88}$, or in lesser tearmes, $1\frac{7}{11}$, and then the fygure wyll stand thus $\frac{1}{1}\frac{1}{2} \angle 1\frac{7}{11}$ $\frac{1}{2}$ — $\frac{2}{4}$
M. These termes now you vnderstand wel ynough: And as for any other at this tyme, I wyll not repeat, onlye thys shalt you marke for the proofe of thys rule, whether your worke be well wrought or no. Multiplye the fyrste numbre by the forthe, and note what amounteth. then multyplye the seconde by the thyrde, and marke what amounteth also. Nowe yf those two numbres so amountyng be equal, then

The proofe of thys rule.

then is your worke well doon, els you haue erred. And this shall suffice for the former rule, but in the Backer rule, thys shal you note for ease of worke, that you multiplye the numeratour of the fyrste, by the numeratour of the seconde, and the whole thereof be the denominatour of the thyrde, and that amounteth thereof, shall be the diuidende. Then multiplye the denominatour of the firste, by the denominatour of the second, and that whole by the numeratour of the thyrde, and that ryseth thereof shall bee the diuisor. Example of thys: I dyd lende my feyende $\frac{3}{4}$ of a Porteguise vij. monethes, vpon promise that he should do as much for me agayne, and when I shoulde borowe of him, he could lend me but $\frac{1}{2}$ of a porteguise now I demaunde how long tyme muste I keepe hys money in iuste recompence of my lone? accomptyng 12 monethes in the yeare? Scholler. The fyrste noumbre muste be the fyrst monye borowed, that is $\frac{3}{4}$ of the Porteguise, the second numbre the 7 moneths, that is $\frac{7}{12}$ of a peare, and the thyrd numbre the mony that was lent in recompense, that is $\frac{1}{2}$ of a porteguise, then I sette the numbres thus.

$\frac{3}{4}$ $\frac{7}{12}$ $\frac{1}{2}$

Then (as you tawghte me) I multiplye 3 (beyng numeratour in the firste numbre) by 7 the numeratour of the second numbre, and it maketh 21, which I multiplye by 12

by 12 the denominatour of the thyrd, and so haue I 252 for the diuidende: then I multiplys 4 the denominatour of the fyrste, by 13 the denominatour of the seconde, and it yeldeth 52, whych I multyply agayne by 5, the numeratour of the thyrde, and yt wyll make 260, that is the diuisour. Then muste I by vyde 252, by 260, so it wyll be in the smallest fraction of a yeare. And this doo you see some ease, in woorkynge better them to multiply and dyuyde tediously so many fractions. An other question yet wyll I propose, to the intent you may see thereby the reason of the statute of assise of bread and ale, whiche in all statute bokes in Frenshe, Latine, and Englysh, is muche corrupted for wante of knowledg in this arte, for the right vnderstandyng whereof I propone this question. When the price of a quarter of wheat is it 8. the farthyng white lofe shall way 68 S, then I demaunde what shall suche a lofe waye, when the quarter of wheate is solde for 28. Sco. Thys question muste be wrought, as yt is proponed in whole noumbres and not in fractions. M. You seeme to say reasonably, howbe it in that statute of assise, the rate is made by the proportion of partes in a pounde weygh Troye, els could it not be a statute of any longe continuaunce, seing the shyllynges do chaung often, as all other monies doo, but this statute wyll vnderstande,

Statute assise of bread & a

is

es a continuall rule for euer, as I wyll anon declare by a newe taule of assise, conuerting the shyllynges into partes and part of partes. Therefore here by a shyllyng you muste vnderstande $\frac{1}{20}$ of a pounde waighte, and so by pens. $\frac{1}{240}$ of a pounde, wherfore although you myghte worke this question proponed by whole nombre well enoughe for that tyme when the Statute was made, yet to apply it to our tyme, and to make it serue for all tymes generally, it is beste to worke it by fractions, setting for 1. shyllynges $\frac{1}{20}$, and for 68 shyllynges, $\frac{68}{20}$ and so for three shyllyngis $\frac{3}{20}$, and then wyll the figure of the question stand thus,

$$\frac{1}{20} \quad \frac{68}{20} \qquad \frac{3}{20}$$

In whych question bycause all the denominatours be lyke, you shall worke only with the numeratours. Scol. Then I shal multiply 68 by 2, whereof commeth 136, whych if I deuide by 3, the quotient wylbe $45\frac{1}{3}$. but howe shall I make a fraction of that to stand with the other? M. Haue you so sone forgotte what was taught you so late lye? Thys is thys fourme. $\frac{45\frac{1}{3}}{20}$

Sc. I remembre it now. and then it signifieth 45 twenty partes, and the third parte of one twenty parte. Maist. Sots it, and that maketh in shyllyngs 45 shyllings, foure pens. wherby you may note one greate errour in the statute bookes which haue constantly 48 shyllings in that assise. And by this

rule

le yf you examin the statute, you shall fynd many summes false, wherefore for the true vnderstandyng of that statute and such like as I haue made mention of it, and somwhat recognised it, so do I wyshe that all gentlemen and other studentes of the lawes, would not neglecte thys arte of Arithmetike as vnprofitable to their studies. Wherefore to encourage them thereto, and to gratifye bothe them & al other in general, I wyl here exhybite a table of that parte of the statute in ii. columnes, and in a third columne I wyl adde the correction of those errours wyche haue crept into it.

Heere foloweth the Table.

When a quarter of wheat is solde for

The price of a quarter of wheat.		The weight of a sterling pounde lost by the statute hooken.			The collection by the Assise.		
s.	d.	li.	s.	d.	li.	s.	d.
1	0	6	16	0	6	16	0
1	6	4	10	8	4	10	8
2	0	3	8	0	3	8	0
2	6	2	14	4 $\frac{2}{5}$	2	14	4 $\frac{2}{5}$
3	0	2	8	0	2	5	4
3	6	2	2	0	1	18	10 $\frac{2}{5}$
4	0	1	16	0	1	14	0
4	6	1	10	0	1	10	2 $\frac{2}{5}$
5	0	1	8	2 $\frac{2}{5}$	1	7	2 $\frac{2}{5}$
5	5	1	4	8 $\frac{2}{11}$	1	4	8 $\frac{2}{11}$
6	0	1	2	8	1	2	8
6	6	0	19	11	1	0	11 $\frac{1}{13}$
7	0	0	19	1	0	19	5 $\frac{2}{5}$
7	6	0	18	1 $\frac{1}{2}$	0	18	1 $\frac{3}{4}$
8	0	0	17	0	0	17	0
8	6	0	16	0	0	16	0
9	0	0	15	0 $\frac{1}{4}$	0	15	1 $\frac{1}{5}$
9	6	0	14	4 $\frac{3}{4}$	0	14	3 $\frac{15}{132}$
10	0	0	13	7 $\frac{1}{2}$	0	13	7 $\frac{1}{5}$
10	6	0	12	11 $\frac{1}{4}$	0	12	11 $\frac{1}{5}$
11	0	0	12	4 $\frac{1}{4}$	0	12	4 $\frac{4}{11}$
11	6	0	11	10	0	11	9 $\frac{22}{132}$
12	0	0	11	4	0	11	4

In ye cõmon bokes ther is no farther rate of assise made thã vnto 12 s ye quarter of wheat, but in an ancient copy of 200 yeres old (whe the I haue) ther is added ye rate of assise vnto 20 s the quarter, but yet was ye assise also ether wrong cast at the first penning, or els corrupt syth ye time for lack of iust knowledg in ye rule of proportiõ, which I will adde here also to gratify such as be studious & this sort of knowledg, & desir to vnderstãd truth exactly

The price of a quarter of wheat.		The weighe of ye farthing white lost by the statute booke.		The correction of the errours.	
s.	d.	s.	d.	s.	d.
12	6	11	0	10	10 $\frac{1}{3}$
13	0	10	5 $\frac{1}{2}$	10	5 $\frac{7}{13}$
13	6	10	1 $\frac{1}{2}$	10	0 $\frac{2}{3}$
14	0	9	7	9	8 $\frac{3}{7}$
14	6	9	2 $\frac{1}{2}$	9	4 $\frac{7}{29}$
15	0	9	1 $\frac{1}{2}$	9	0 $\frac{4}{5}$
15	6	9	1 $\frac{1}{2}$	8	9 $\frac{9}{31}$
16	0	9	0	8	6 0
16	6	8	6	8	2 $\frac{30}{33}$
17	0	8	3	8	0 0
17	6	7	10	7	9 $\frac{2}{35}$
18	0	7	6	7	6 $\frac{1}{3}$
18	6	7	3	7	4 $\frac{3}{37}$
19	0	7	2	7	1 $\frac{17}{19}$
19	6	6	10	6	11 $\frac{9}{39}$
20	0	5	6	6	9 $\frac{1}{5}$

Theſe. ii. tables I haue ſet ſeuerall, becauſe no man ſhoulde thynke that I would either adde or take away from any lawe, thoſe partes whiche myght of ryght ſeme eyther ſuperfluous other diminute, but yet I maye not be ſo curious as to neglect manyfeſt errours, whiche is not onely my parte, but euery good ſubiectes duetie, with fodzietie to correcte. And for auoydynge of offence, I haue rather doone it in this priuate booke, rather then in any boke of the ſtatutes ſelfe, truſtynge that all men wyll take it in good parte. Scholar. I woulde wyſhe ſo, but I dare not hope ſo, ſyth neuer good man that woulde reſourme errour, coulde eſcape the venymous tungues of enuious detractors, which becauſe they either can not or liſt not do any good theim ſelues, do delite to barke at the doynges of other, but I beſeke you to ſtay nothyng for their pueriſe behauiour. M. I conſyder many thynges that ſom may obiect, whereunto I am not vnprouided of ſuche aunſwers, but I wyll not ſeeme ſo haſty to make the anſwers before I heare their obiections, but as I truſt that men are of a better nature, and more gratefull nowe, then ſome hath ben in tymes paſſed, as I haue doon in the ſtatute of aſſiſe for bread in rate of ſhillynges, ſo will I ſet forthe the like table in poundes & vnces, and the partes ther of, that it may be eaſily applied to al tymes,

but

but I meane not by this to alter any worde of the statute (beynge so good an ordinance and of so great cõtinuance) but only to make it as a kind of exposition and declaration of the said statute, trustyng that therby the statute may be better vnderstand, & consequently better put in execution. And here you shal note that I haue accompted the shyllynges after the rate of. ix. s. to the pounde weyght, because I esteeme it the most apte rate for our tyme. Wherfore in the fyrste columne you fynde the price of wheate, directly agaynste it in the seconde columne you may fynd the weyght of the farthyng white lofe, in this our tyme, and if you double that numbre (as I haue doone in the thyrde columne) thenne haue you the weyghte of the halfe penny white lofe, and so in the fourth columne is sette the weyght of the peny wyhte lofe. It needeth not to tell you that, that the fyght dooth testify, how that euery columne is parted into thre smaller pilers, whereof the fyrste columne hathe these titles, poundes, shillynges and pennes: the other three columnes, haue eche of theym these three titles, poundes, vnces, and peny weyghtes. And as in the fyrst columne xii. pence make a shillyng, and. xx. s. maketh a pounde, so in the other. iii. columnes xx. peny weight maketh an vnce, and xii. vnces doo make a pounde.

quarter of wheate.

li.	s.	d
0	3	0
0	4	6
0	6	0
0	7	6
0	9	0
0	10	6
0	12	0
0	13	6
0	15	0
0	16	6
0	18	0
0	19	6
1	1	0
1	2	6
1	4	0
1	5	6
1	7	0
1	8	6
1	10	0
1	11	6
1	13	0
1	14	6
1	16	0
1	17	6
1	19	0
2	0	6

ripping wheate lose.

li	bucc	d. w.
6	9 1/2	2
4	6 1/2	3
3	4 3/4	1
2	8 1/2	2 1/2
2	3	4
1	11 1/4	1 1/2
1	8 1/4	3
1	6	2 3/4
1	4 1/2	1 1/2
1	2 3/4	1 1/8
1	1 1/2	2
1	0 3/4	1 1/3
0	11 1/2	3 1/2
0	10 3/4	2 1/2
0	10	4
0	9 1/2	2
0	9	1 1/2
0	8 1/4	1 1/10
0	8	3 1/2
0	7 3/4	0 3/4
0	7 1/2	2 1/3
0	7	1 1/2
0	6 3/4	1
0	6 1/2	0 4/5
0	6 1/4	0 2/3
0	6	0 7/8

halfpeny wayght lofe.

li.	vnc.	ß. m.
13	7	4
9	0 1/2	1
6	9 1/2	2
5	5 1/2	0 1/2
4	6 1/2	3
3	1 0 1/2	2 1/2
3	4 1/2	1
3	0 1/2	0 1/2
2	8 1/2	2 1/2
2	5 1/2	3 1/2
2	3	4
2	1	2 1/2
1	11 1/2	1 1/2
1	9 1/2	0 1/2
1	8 1/2	3
1	7	4
1	6	2 1/2
1	5	3 1/2
1	4 1/2	1 1/2
1	3 1/2	0 1/2
1	2 1/2	1 1/2
1	2	3 1/2
1	1 1/2	2
1	1	1 1/2
1	0 1/2	1 1/2
1	0	1 1/2

ng wayght lofe.

li.	vnc.	ß m.
27	2 1/2	3
18	1 1/2	2
13	7	4
10	10 1/2	1 1/2
9	0 1/2	1
7	9 1/2	0 1/2
6	9 1/2	2
6	0 1/2	0 1/2
5	5 1/2	0 1/2
4	11 1/2	1 1/2
4	6 1/2	3
4	2	4 1/2
3	10 1/2	2 1/2
3	7 1/2	0 1/2
3	4 1/2	1
3	2 1/2	3
3	0 1/2	0 1/2
2	10 1/2	2 1/2
2	8 1/2	2 1/2
2	7	1 1/2
2	5 1/2	3 1/2
2	4 1/2	2 1/2
2	3	4
2	2	2 1/2
2	1	2 1/2
2	0	3 1/2

Scholar. Syr I doo thanke you moste hartily, for this not onely in myne owne name, and in the name of all studentes, but also in the name of the hole cõmons, to whom the restitution of this assise (I truste) shal bryng restitution of the weygot in breade, whiche longe tyme hathe been abused. And yf you knowe any lyke thynges more, wherin you would vouchsafe to declare the errours, and sette forth the truthe, you can not but obtayne great thankes of all good harted men that loue the common wealthe. M. I haue sundry thynges to declare but I haue reserued them for a priuate booke by it selfe, yet not withstandyng bicause the statute of the rate of mesuryng of grounde is so common that it toucheth all menne, and yet no more common thenne nedefull, but so muche corrupt, that it is to farre out of all good rate, not onely in the englishe bookes of statutes, commonly printed, but also in the latin bookes, and in the frenche also, for I haue read of eche sorte, and conferred them diligently. I wyll gyue you a table for the restitution of those errours, as may suffice for this presente tyme. And syrste wyll I propose one question to you, touchyng the vse of that statute, whereby you maye perceiue the order howe to examyn the whole statute, and euery partell thereof, and the question is this, When the aker of grounde booth contayne

foure

foure perches in bredthe, than muste it conteyne 40 perches in lengthe, thenne doo I demande of you, how much shall the length of an aker bee, when there is in the bredthe of it 13 perches? But before you shall answeare to this question, I wyll declare vnto you an other statute, whyche is the ground of that former statute. And that statute is this, It is ordeined, that 3 barly cornes dry and rounde shall make vp the measure of an ynche, xii. ynches shal make a foote, and iii. foote shall make a yard (the common englysshe bookes haue an eine) fyue yardes and halfe shall make a perche, and forty perches in lengthe, and. iii. in bredthe shall make an aker: this is that statute, whereby you may perceaue, that the intente of the statute is, that an aker shoulde conteyne 160 perches. Nowe lette me here youre answeare to the question. Scholar. As I perceaue by the wordes of that statute, a perche to bee $\frac{1}{160}$ of an aker, so could I make those numbres all in fractions, and so worke the question, but seyng I maye doo it also in hole noumbres, I take that forme for the moste easy, therefore thus I sette the question in forme, 4 Z 40 then doo I multiply 13 40 by 4, and it maketh 160, whyche I diuide by 13, and the quotiente is 12 $\frac{4}{13}$ M. Nowe turne that $\frac{4}{13}$ into ꝑ cõmon ꝑtes of a perche, as they be na

A statu measur

med in the former statute, howbeit it shall be beste to take one of the least partes in denomination for anoydyng of muche labour, as feete, wherof the pearche contayneth 16½. Scholar. Than to tourne $\frac{4}{11}$ into feete, I multiply 16½ by 4, and it maketh 66, whyche I must diuide by 11, and the quotient is $5\frac{1}{11}$. Maist. So you fynde that if the acre holde in bredthe. xiij. perches, it shall conteyne in length 12 perches, 5 foote, and $\frac{1}{11}$ of a foote, whyche is not fully an ynche, for the ynche is $\frac{1}{12}$ of a fote. but here all the statute bokes in latin and englysh (that I haue sene) do note it to be 13 perches, 5 foote 9 inch, whyche maketh aboue. xiij. perches to many in the aker, so that I woulde haue thought the errour to haue crept into the printed bokes by the great negligence that printers in our tyme do vse, saue that in written copies of great antiquitie, I doo fynde the same. Yet haue I one frenche coppe, whyche hath 12 perches ¼, and one foote, and that mysseth very lyttell of the truthe. Scho. Then I see it is trewe, that I haue often hearde saye, that the trewest coppes of the statutes bee the frenche coppes. Mayster. That is often trewe, but not generally, as I haue by conference tryed dyuersely, but in this statute the frenche booke is moste corrupte in all other places lyghtly.

But nowe to performe my promyse, I

wyll

wyll sette foorth the table for measurynge of an aker of grounde onely by suche partes as the statute doothe mention, bycause at this tyme I doo of purpoke wryte it for the better understandyng of the statute, and hereafter, with other thynges, I entende to sette foorthe this same more at large.

In this table folowyng I haue not done as in the other statute before, compared my restitution with the faultes crept into the statute, but onely haue written that trew measure, which the equitie of the statute doothe pretende, for it were to vyle to iudge of so noble pryntes and worthy counsellours as haue auctorised and set foorth this statute, that they wold make one aker in any forme greater then an other, but euery one to be iuste and equall with eche other, whyche is the grounde also of my worke, and hereby may all menne perceyue howe needefull Arithmetike is vnto the studentes of the law. But now I thynke beste to make an ende of these matters for this present tyme, syth the table hathe in it none obscuritie, that I shoulde needs to declare.

The breth the	The length of the aker.	
perch	perch	feete
12	16	10
13	12	9
14	11	5 1/2
15	10	6 1/2
16	10	0
17	9	6 2/7
18	8	14 2/3
19	8	6 4/9
20	8	0
21	7	10 1/2
22	7	4 1/2
23	6	15 2/3
24	6	11
25	6	6 3/5
26	6	2 7/5
27	5	15 5/8

The brede the	The length of the aker	
yches	nch	feete
28	5	10 1/2
29	5	8 2/8
30	5	5 1/2
31	5	2 4/5
32	5	0
33	4	14 2/3
34	4	11 1/2
35	4	9 3/7
36	4	7 1/2
37	4	5 2/3
38	4	3 9/10
39	4	2 2/3
40	4	0
41	3	14 4/5
42	3	13 1/2
43	3	12 2/5
44	3	10 1/2
45	3	9 1/6

Scholar. In dede sir, I vnderstande the table (as I thynke) by those other whiche you sette foorth before. For in the fyrst columne is sette the perches of the brede of any aker

ker, and then in the. ii. columnes folowyng appereth howe many perches, and how many foote the same aker muste haue for his lengthe. Maister. You take it well, howbeit to speake exactely of breadth and lengthe, the fyrste columne dooth sometyme breoken the breadth, and sometyme the lengthe, for properly the longest syde of any square, doeth limytte his lengthe, and the shorter syde dothe breoken the breadth, yet it is no greate abuse in suche tables where a man can not well chaunge the title, to lette the name remayne, although the proportions of the numbres doo chaunge. for styll by the fyrste columne is expressed the measure of the one syde, and by the. ii. other pillers in one columne, is set forth the measure of the other side. And this shalbe sufficiēt now for the vse of the golden rule. Nowe somewhat will I touche certayne other rules, which for their seuerall names may seme diuers rules and distincte from this, but in deede they are but braunches of it, yet bicause thei haue not only seuerall worckyngs in aperance, but also pleasant in vse, I will geue you a taste of eche of them. As for the rule of felowshyp both single and double, with tyme, & without tyme, I shall neede to say lytle more than I haue all ready sayd in teachyng the woorkes of hole numbres, yet an example or. ii. wyll we haue to refreshe the remembrance of the same,

and

and to declare certayne profytes vses and ap-
plicatiõs of it, as this for one. Foure mẽ gat
a booty or pryse in tyme of warre, the pryse
is in value of money 8190 li, and because the
men be not of lyke degree, therfore there sha-
res may not be equall, but the chiefest person
will haue of the bootie the third parte, the .x.
part ouer the second will haue a quarter and
the .x. part ouer, the third will haue the .vi.
part, and so there is left for the fourth man
a very small portion, but suche is his lotte.
(whether he be pleased or wroth) he must be
content with one .xx. part of the pray. Nowe
I demand of you, what shal euery man haue
to his share? S. You must be fayn to answer
to your owne question, els it is not like to be
answered at this tyme. M. The forme to vn-
derstand the solution of this question, and all
such like is this. reduce al the denominators
into one nũbre by multiplicatiõ, except that
any of them be partes of som other of them,
for all suche partes you may ouerpasse and
take for them all those numbres whose par-
tes thei bee, as in this exaumple, the shares
bee these, $\frac{1}{3}$ $\frac{1}{10}$, $\frac{1}{4}$ $\frac{1}{10}$, $\frac{1}{6}$, $\frac{1}{20}$, yf I multi-
ply all the denominatours togither, begyn-
nyng with 3, and so go on vnto 20, it wyll
make 144000, but consyderyng that 3 is a
part of 6, I shall omit that 3, & likewais 10
which is a part of 20, I may ouerpasse also,
and then is there but 3 denominatours to
multi-

question
vnequal
cties.

multiply, that is 4, 6, and 10, whiche make 240, which summe I take for my worke, bycause al the denominators will be found in it. Then I take suche partes of it as the question importeth, that is for the first man, $\frac{1}{4}$, $\frac{1}{10}$, the $\frac{1}{4}$ is 60, the $\frac{1}{10}$ is 24, whiche I put in one summe for the first mās share, and it maketh 104. Then for the second mās share I take $\frac{1}{4}$, whiche is 60, and $\frac{1}{10}$ whiche is 24, and that maketh in the whole 84. And for the third man which must have $\frac{1}{3}$, I take 40. And for the fourth mā there remayneth but 12, which is $\frac{1}{20}$ of the hole sum, so that if the hole pray had be but 240 li, then were the question answered, but bycause the summ was of greater value, by this meanes know shall I know the partition of it, I must set my numbers by the order of the golde rule, puttyng in the first place the numbre that I found by multiplyeng the denominatours, and in the second place the sum of the booty. And loke what proportion is betweene that first numbre & that second, the same proportion shall be betwene the partes of the first numbre and the partes of the second, cōparyng eche to his like, therfore I must put in the third place, one of the partes or shares, & then worke by the former rule of proportion or golden rule. And bycause I haue 4 seuerall partes, of the first numbre, by which I wolde fynde out 4 like partes of the second

The rea of this rule.

sum

numbre, therfore must I make 4 seuerall figures. S. Now I trust I can answer to your question, as by your fauour I will proue.

a

240 — 8190
104 — 1365

b

240 — 8190
84 — 2866 ½

c

240 — 8190
40 — 3549

d

240 — 8190
12 — 409 ½

And to try it, I set the 4 figures thus, marked with a, b, c, d, to shewe their order. And then in eche of them I multiply the seconde numbre by the thyrde, and diuide their totall by the fyrste, and so amounteth the fourthe summe; which I seke for, so if I do multiply 8190 by 104, it maketh 851760, which being diuided by 240, maketh in the quotient 3549 for the firste mans portion. And so working with the other 3 figures I fynd for the seconde mā 2866 ½. & for the thyrde mā 1365. And then for the fourth man 409 ½. And so is euery mans share set foorth in the figure here annexed.

a

240 — 8190
40 — 3549

b

240 — 8190
84 — 2866 ½

c

240 — 8190
104 — 1365

d

240 — 8190
12 — 409 ½

And thus I truste I haue doone well. Master. If you multiplye your workinges and lye to proue, aboue all the shares toge-

ther,

ther, and if they make the total, then saieth it well done. Scholar. I may set them thus, and thenne by addition the iuste summe of the amount, that is 8190, & therfore (as you say) it seemeth to be wel wrought. But I beseke you, is there any doubte in this triall, that you vse that words, seemeth? Mai. You may easyly coniecture, that if you dyd assigne the fyfte mans share to the laste, and so change all the rest, that one had an others share, yet would the addition apeare all one, and therfore is not the proofe exacte. But if you wyll make a iuste proofe, for the first mans part take $\frac{1}{10}$ of the hole summe, and if it agree with the noumbre in the figure, then it is well doone. And so doo for the seconde, thirde, and fourth summes, and this proofe fayleth not. Nowe wyll I propounde certayn other questions, whiche haue ben set forth by certayne learned men, all be it not without some ouersight, whiche questions (I protest hartily) I doo not repeate to depraue those good men, whose labours & studies I much praise, & greatly delite in, but only (accordyng to my profession) to seke out truthe in al thynges, & to remoue all occasiõs of error as much as in me lieth, & for that cause I wil only name ỹ questiõs without hurting the autors name. The first question is this. Foure men did build a house

3549	
2866½	
1365	
409½	
8190	

The proofe by addition.

The iuste proofe.

which cost them 3000 crounes, their shares were such, that one man shulde pay $\frac{1}{3}$ of the summe, and 6 crounes ouer, the secõd shuld pay $\frac{1}{4}$ & 12 crounes ouer, the third man must lay out $\frac{2}{5}$, abatyng 8 crounes, & the fift man shuld pay $\frac{1}{4}$ & 20 crounes more. can you answer to this question? M. I am in good faith sir, & that you know best of any mã, for I know no more then you haue taught me. M. Yea I dare say, you can not do it, nother yet the best lerned man that euer dyd propose it. for the question is impossible, for declaration wherof I wil be bold to vse first the representation of the numbres in their apteste forme (although I haue not yet taught you that maner of worke) bicause it maye appere plainly y the question is not possible, for here I haue set the partes, & added thẽ, & then make the hole summ, & $\frac{1}{4}$ and 30 more. Nowe howe is it possible to deuise truely either gaynes either charges so, that the particulars shalbe more then the totall? S. It is against the forme of proffe by addition of the partes. M. you say truth. And bicause you shal perceiue it the better, I will repete after the vulgar forme, as in this figure you se, where the $\frac{1}{3}$ with 6 ouer, is 1506, for the totall is as you heard before 3000, & $\frac{1}{4}$ & 12 more is 1012, the $\frac{2}{5}$ wouldbe 1200,

$\frac{1}{3}$ + 6	1506
$\frac{1}{4}$ + 12	1012
$\frac{2}{5}$ — 8	1192
$\frac{1}{4}$ + 20	792
1 $\frac{5}{6}$ + 30	5290

but

but nowe examyn you this seconde position.
Scholar. If he playde 16 daies, then abatyng 16 tymes 30 ü, the sum will be 480 ü. And for 24 daies that he wrought, euery day yeldyng 25 ü, the totall is 600 ü, so that abatynge 480 out of 600, there remeth 120, and (as you say) it shuld be but 65, therfore is it to muche by 55, that must be set on the right hand of the figure, at the nether part, and ouer it on the same side 16, which is the seconde position thus.

And as I gather by your wordes, it were all one, if I dyd set 28 in stede of 12, and 24 in stede of 16. M. So wer it. But this shal you mark, that of what nature so euer the .ij. positions be, of ỹ same nature is the quotient. Therfore when the positions in this question ar 12, & 16, which bothe bee numbres of the playeng dayes, the quotient shal declare ỹ true nũbre of the playeng daies, where as if the positions had ben 28 and 24, which are supposed to be the worckyng daies, then wold the quotient declare the true numbre of the worckyng dais, and not of playenge daies, as it wyll dowe nowe. And therfore to continue the worcke of this question, and to fynd the true numbre of playeng dais, I must multip!y crosse waies, the fyrst position 12 by 55, that is the seconde errour, and the totall wyll be 660,

then I multiplye 275 by 16, and it yeldeth 4400. Nowe bycause the errours are lyke, that is to say, both to much, I muſt ſubtract 660 out of 4400, and ſo remaineth 3740 whiche is the diuidend. Agayn I muſt ſubtract the leſſer errour 55 out of 275 that is the greater errour, and there wyll remayne 220, whiche ſhall be the diuiſour: then diuidyng 3740 by 220, the quotient wyll be 17. Wherfore I ſay now conſtantly, that 17 is the true nũbre of dais, that the maſon plaied and then it ſoloweth, that he wrought 23 daies, and ſo is the queſtion aunſwered.

The proſe of this rule

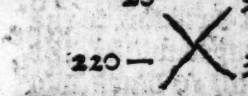
Now for the order of tryall of this worke, there nedeth none other proſe but only this, to worke with this numbre accordyng to the queſtion, and if it agree, then appeareth the numbre to be it that you wold haue. As here nowe ſeyng he wrought 23 daies, and muſt haue for euery day 25 pens, the hole ſumme cometh to 575. Then again ſeyng he plaied 17 daies, and muſte abate 30 pẽs for euery day, the hole ſumme of the abatement wyll be 510. Therfore I ſubtract 510 out of 575 and there wyll remayn 65, whiche maketh 5 ſ 5 d, the clere wages of the maſon for his worke, accordyng to the queſtion. Scholar. Now I truſt I vnderſtãd the work and the rule ſo wel (and the better by this proſe) that I can be able to do the like. And for a proſe I take ỹ ſame queſtion al ſaue the laſt nũbre

where

where I wil suppose that he had 10 s. for his wages clere. And nowe to gesse at the nombre of the dais that he wrought, I suppose first that he wrought 20 daies, then say I, If he wrought 20 dais, his wages must be 500 d. then did he play other 20 daies, for whiche muste bee abated 600 d, and then he leseth 100 d. And so am I at a stay, for it is not like to your former worke. M. you shuld haue required of me some question, and not haue taken a question of your own fantasieng, vntil you wer more expert in this art: for so might you as wel happen on an impossible questiō as on a possible. but now to go forward, consider that this numbre is to little by 220, seyng he shuld gayn by your suppositiō 120 d and in this position he leseth 100, those both make 220, whiche you shall set down for the first errour, with this signe —— betokenyng to litle, as here in the forme folowyng doeth appeare. And nowe for the rest, go forwarde your self ones agayn. Scholar. As my errour hath vttered my foly, so it hath procured me better vnderstādyng. Nowe therfore considderyng this position not to solue the questiō, I take an other, supposyng that he wrought 30 dayes, then for his wages he must be allowed 750 d, and for the 10 dais which he wrought not, he muste abate 300 d, and so remayneth clere 450 d.

20
220—

but it shuld be onely 120 ℔, therefore is it to muche by 330, whiche I sette downe in the figure with the former position and his errour, & the figure apereth thus, as folowe.



Now must I multiply in croise waies 220 by 30, and it wyll bee 6600, Then agayne I multiply 330 by 20, and it wyll be also 6600. Wherfore if I shall subtract the one out of the other, there will remayn nothig to be the diuidend. M. In this you forget your selfe agayne, for in as much as the signes of the errours be vnlike, therfore must you worke by addition, addyng to gether those. ij. totals to make the diuidēd, and addyng also the. ij. errors, to make the diuisour. And bycause you shall no more forget this part of the rule, take this breefe remembrance.

Vnlyke requyre Addition,
And lyke desire Subtraction.

Scho. You meane that if the errours haue lyke signes, then must the diuidend and the diuisor be made by subtractiō, as is taught before, and if those signes bee vnlyke (as in this laste example they be) then muste I by addition gather the diuidende and the diuisour. Therfore must I adde 6600 to 6600, and it wyll be 13200, whiche shall be the diuidende. Then agayne I adde 220 to 330,

and

and it will be 550, whiche muſt be the diuiſour, wherefore diuidyng 13200 by 550, the quotient will be 24, whereby I knowe that the maſon wrought 24 dais, and then it followeth that he playd 16 daies. M. Examyn your worke whether it bee agreable to the queſtion or no. S. For 24 dayes worke, the wages muſt be 600 d, and for 16 daies, whi che the maſon wrought not, there muſt be abated 480 d, and then remayneth cleere to the maſon 120 d. as the queſtion importeth, wherefore it is euident that 24 is the trewe numbre of the days that he wrought. M. Although you ſeeme nowe to vnderſtande this worke, yet to acquaynt your mynde the better with the new trade of this rule, I thynk it good to propone to you. iiij. or iiij. exãples more before I make an ende of it. S. Syr I thanke you that you do ſo conſider my commoditie and profit in knowlege, for vndoubtedly it is practiſe and exercile that maketh men prompte and experte in euery kynde of knowlege. M. You ſay well, ſo that they followe ſome certayn preceptes to gouern and rule their practiſe by, els may practiſe procure cuſtome of errour, and a repugnance to exactnes of knowlege, namely as longe as the error is not playnly knowen to the vulgar ſort. But to return to our worke, There is a ſeruant that hath bought of veluet and damaſke for his maiſter 40 yardes, the vel-

uestion
wares

ket at twenty shillynges a yarde, and the da-
maske at 12 ℔, and when he cometh home,
his mayster demandeth of hym howe much
he hath bought of eche sort, I can not tell
(sayth he) exactly, but this I knowe, that I
payde for damaske 48 ℔ more then I payd
for veluet, now must you gesse howe many
yardes there is of eche sort. Sco. Although
the gesse semeth difficulte, yet I wyll proue
what I can doo, for I remembre your say-
eng, that it forceth not howe sonde or false
the gesse be, so it be somewhat to the questiō
and not an answere of a contrary mattier.
Therfore fyrst I imagyn that he bought 20
yardes of damaske, for whiche he shuld pay
after the former price 240 ℔, then must he
nedes haue of veluet other 20 yardes (to
make vp the 40 yardes) and that wold cost
400 ℔. So that the totall of the price of the
damaske is lesse than the summe payde for
veluet 160 ℔, and shuld be more by 48, ther
fore the fyrste errour is 208 to little. Then
begyn I agayne, and suppose he bought of
damaske 30 yardes, that coste 360 ℔, then
had he but 10 yardes of veluet, whiche coste
200 shillynges, and nowe the price of da-
maske is greater thē the price of ỹ veluet by
160 shillinges, and shoulde bee but 48, there-
fore is the seconde errour 112 to muche,
whyche I sette in fourme of a figure, as
here doothe appeare. Then doo I multi-

plye

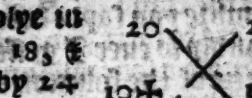
ply in crosselwaies 208 20 30
by 30, and the summe
will be 6240. Also I
multiply 112 by 20, & 208— 112+
there wyll amount 2240. And in as muche as the signes of the errours bee vnlyke, I knowe I muste worke by addition, therfore adde I those .ij. totals togyther, and they make 8480, whyche is the dividende, then adde I also the .ii. errors togither 208 and 112, and they make 320, whyche is the diuisour. wherfore diuidynge 8480 by 320, the quotient wyll be 26 ½, whyche is therrewe summe of yardes of damaske that he bought and in velvet 13 yardes ½, and that apereth by examination, thus: 26 ½ yardes of damaske at 12 s. the yard, maketh 318 s, then in velvet he had but 13 yardes and ½, that cost 270 s, at 20 s. the yarde. Now subtract 270 out of 318, & ther wil remain 48, which is the nombre of shillings that the damaske dyd cost more then the velvet. M. Now shall you haue a question of an other kind. There ar iij. men that do owe money to me, & I haue forgotten what the totall summe is, and also what the particulars be. &c. Why? then it is impossible to knowe the dette. M. Peace ye are to hasty, there is more helpe in it then you yet se, I haue 3 seueral notes, wherby it apereth that I dyd confer theyr debtes togither, and found the dette of the first man

A question of debt.

and the second to amount to 47 li, the debt of the fyrst man and the thyrd, dyd make 71 li, and the seconde man his debte with the thyrde, dyd ryse to 88 li. Nowe can you tell what euery man dyd owe, and what was the hole totall? Scho. Nay in good faith, but as I perceaue that it must be founde by coniecture, so will I gesse at it, supposyng that the fyrst man did owe 20 li, and the second man 30, and the thyrd Mai. Nay stay there, for you are to farre gone all ready, you maye not suppose a seuerall summe for euery man for it is ynoughe to suppose one summe for the fyrst man, and lette the other ryse as the question importeth. Therfore seyng you set the fyrst man his debt to be 20 li, the second man can not owe 30 li, for the declaration is, that theyr debtes added togyther dydde make 47 pound, so must the seconde man his debte bee but 27 li. Nowe this seconde debte with the thyrde muste make 88, therefore subtracte 27 out of 88, and there wyll remayne 61, as the thyrde man his debte. Then sayth the declaration that the fyrst and thyrd mens dettes doo make 71, but by this supposition they make 81, that is 10 to muche, whiche I must set for the fyrst error. Nowe worke you the seconde position. Sco. I suppose the fyrst mans debt to be 24 pound then must the second mans debt by your declaration be but 23 li. seyng bothe they doo

make

make but 47 li. Also the seconde man his dette with the thyrd doo make 88 li. and the second man owerth but 23, therfore the thyrd man must owe 65 li. Now the thyrd mans debt with the fyrst shuld make by the declaration 71 li. and they doo make 89 pound, that is 18 li. to muche, and that is the seconde errour, whiche I set downe with the fyrst. and their positions in this forme, and then doo I multiplye in crossewaies 20 by 18, & it is 360. Also 10 by 24 maketh 240. And by-



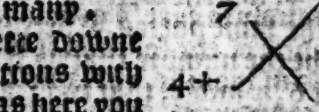
cause the signes of the errours bee lyke, I must worke by subtraction, therefore I subtract 240 out of 360, and there resteth 120, whiche is the diuidend, then doo I subtract 10 oute of 18 by the same reason, and so is the diuisour 8, whiche is founde 15 tymes in 120, therfore I say that the fyrste man byd owe 15 li, and then the seconde man muste owe 32 li, for those two do make 47 li. and the thirde manne his debte is 56, for so much the remaineth, if I bate 15 out of 71, or yf I take 32 out of 88. Mat. For the third example take this easy question, for the varietie in worke. Two men bayyng seueral summes, whiche I knowe not, doo thus talke together, the fyrst sayth to the second, If you geue me 2 of your money, than shal I haue iii. tymes so much money as you; the second

The 6 question

numbereth, it were more reason, that both summes were made equall, and so will it be, if you geue me 3 s. of youre money. Nowe gesse what eche of them had. S. I imagine that the first had 9 shillings. M. Conlyder euermore in youre imaginations that you take a likely summe, as in this question take such a summe that hauyng 2 added vnto it, may be diuided into 3 partes euen. S. Why I remembre you sayde before, it forced not, howe fondly so euer I gessed. M. As for the possibilitie of the solution it is truth, but for easynes in woorke, the aptest numbres are moste conuenient. Scho. I thought no lesse, and therfore I tooke 9 as an apte noumbre to be parted into 3. But I perceaue I shuld haue conlydered the aptnesse of that partition after the addition of 2 vnto it, and then 7 hadde ben more meter. Mayster. That is truthe, and then should the second man his summe be 5, for although he haue nowe but the thirde parte of 9, that is 3, yet you must remembre that he leuie the fyrste manne 2, and so had he 5. Scholar. Then to go forewarde, if the seconde man had 3 of the firste man, then shoulde he haue 8, and the fyrste man but 4, so hath he double to the first mā, yet he sayd in the question, they shulde haue equall, wherfore it appeareth that he hathe 4 to muche. Therfore I note that errour with his supposition, and gesse agayne that

he

he hathe 10 shillynges, wherevnto I adde two shillynges, borowed of the seconde man, and then hathe he 12 shillynges, so the seconde manne hath remaynynge but towe, wherevnto yf I adde the 2 that he lente to the fyrste manne, so hadde he but 6 s̄ at the begynnyng. Then take 3 shillynges from the fyrst man, and gyue to the seconde, then hathe the fyrste manne but 7, and the seconde hath 9, whiche are not equall, but there are 2 to many. wherefore I sette downe bothe the positions with their errors, as here you see, and multiply a crosse, so commeth there 40 and 14, and bicause the signes be vnlike, I take 14 out of 40, and so resteth 26 to be the diuidende: then lykewaies I take 2 out of 4, and there resteth 2, by which I diuide 26, and the quotient wyll be 13, whiche is the summe that the fyrst man hadde. And so appeareth that 2 beeyng added thereto, the summe wyll be 15, so hath the seconde man nowe but 5, and before he had 7, then take 3 from the fyrste, and putte to his seuen, so haue eche of theym tenne, and that is equall, as the questyon woulde. Maysters.



For the fourthe exeumple take this questyon, One manne sayde to an other, I thynke you hadde this yeare twoo thousande lammes, so hadde I, sayd the other,

The fourthe exeumple

bnt

but what with payeng the tythe of theim, and then 3 seuerall losses, they are muche abated, for at one tyme I losse halfe as many as I haue now left, and at an other tyme the thyrde parte of so many, and the thirde tyme $\frac{1}{4}$ so many. Now gesse you how many are lefte. Sco. Bicause here is mention made of certayn partes, I must take a nombre that may haue all those partes, that is to saye, $\frac{1}{2}$ $\frac{1}{3}$ and $\frac{1}{4}$, whiche wyll bee 24, howebeit 12 hath the same partes. Therfore fyrst I take 12 to be the numbre that do remayn, so hath he lost 6, 4 and 3, that is 13, and in the hole 25, but it shuld be 2000. M. ye are deceiued yet styll, you haue forgotten the tenth part, which must be defalked, that is 200, so there remaineth but 1800: And now go on again. Sco. Then to fynde the errour I take 25 out of 1800, and there remaineth 1775 to few, whiche I set for the fyrste errour. Then for the seconde position I take 24, whose halfe is 12, the thyrde parte 8, and the quarter 6, wherby riseth 50, which is to litle by 1750, therfore I set downe bothe the positions with theyr errours thus. And multiplye in crosse wayes 1775 by 24, wherof cometh 42600. Also I multiply 1750 by 12, & there riseth 21000. And bicause the signes ar like, I do subtract the one from the other, and so remai

12 — 24

1775 — 1750

remaineth the diuidende 21600, then do I
subtract 1750 out of 1775, and ther resteth
25, by whiche I diuide 21600, and the quo-
tient is 864. whereof the halfe is 432, and
the thyrd part is 288, the quar-
ter is 216, whiche all beyng ad-
ded togither wil make 1800. And
if you adde therto the tenth whi
che was abated before, then wil
the hole sum be 2000. And nowe
dothe there come a question to my memory,
whiche was demaunded of me, but I was
not able to answere to it, and nowe me think-
eth I coulde solue it. Mai. Propone youre
question. Scolar. There is supposed a lawe
made (for furtheryng of tyllage) euery ma
that doth kepe shepe, shall for euery 10 shepe
eare and sowe one acre of grounde, and for
his allowance in sheepe pasture, there is ap-
pointed for euery 4 shepe one acre of pasture.
Nowe is there a ryche shepemaister whyche
hath 7000 akers of grounde, and woulde
gladly kepe as many sheepe as he myght by
that statute, I demaunde howe many shepe
shall he kepe? Mai. Answere to the question
your selfe. Scho. Fyrste I suppose he maye
kepe 500 shepe, and for them he shall haue in
pasture, after the rate of 4 shepe to an acre,
125 akers, and in earable grounde 50 acres
that is 175 in all, but this errour is so litell
by 6825. Therefore I gesse agayn that he

8 6 4
4 3 2
2 8 8
2 1 6
1 8 0 0

A questi
of shepe
tyllage.

maye

may kepe 1000 shepe, that is in pasture 250 akers, and in tyllage 100 akers, which maketh 350, that is to lytle by 6650.

500 1000
6825— × 6650—

These bothe errounes with theyr positions I sette downe as you see, and multiply in crosse 6825 by 1000, and it maketh 6825000. Then I multiply 6650 by 500, and it doth amount to 3325000, which sum I do subtract out of the fyrst, & there remaineth 3500000, as the diuidende. Also I doo subtract the lesser errour out of the greater, and so remayneth 175, by whyche I diuide the said diuidende, and the quotient will be 20000, so that I se that by this rate he that hath 7000 acres of grouñd may kepe 20000 shepe, & therby I coniecture that many men may kepe so many shepe. for many men (as the common talke is) haue so many acres of grouñd. M. That talk is not likely, for so muche grounde is in cõpas aboue 48 ¼ miles. but leaue this talke, & returne to your questions, least your pointing be scarse wel take. S. In dede I doo remẽbre ỹ the Egyptians did grudge so muche against shepardes, till at length they smarted for it, & yet they were but small shepemaisters to some men that be now, and the shepe are waxen so fierce nowe and so myghtye, that none can withstande them but the lyon. M. I perceaue you talke

as

as you here some other, but to the worke of your questiō, both this last question and the next before might be wrought without the secōd position, by the rule of proportion, as this: Whā in this question you found in the first errour, that for 500 shepe, there must be 175 acres, then myght you reduce it to the golden rule, thus: If 175 acres will admyt in allowance 500 shepe, 175 ⅄ 500 than 7000 wyll haue 7000 ⅄ 20000 20000. And so by one positiō with the helpe of the golden rule may you answer that question. Likewais for the question of lambes, when you had found that 12 came of 25, you myght haue set the figure thus as you see, and haue sayd, If 25 do leaue but 12, what shall 1800 leaue? and it would appeare to bee 864. Scholar. Syr I thanke you for this ayde, for it doothe muche shorten the woorke of this rule. Maister. Yet agayn wyll I shewe you an other waye to aunswere to this laste question without this rule of false position, and that by the rule of felowshyppe, for it appearethe in the proponynge of the question, that tenne sheepe muste haue in pasture 2 akers and ½, and for theym muste there bee eared but one aker, so it followeth, that for two akers eared there muste bee fyue sette to pasture. And yf you putte theym bothe in to oone Summe,

An other waye in woorkyng

25 ⅄ 12
1800 ⅄ 864

An other way yet

they

they wyll make 7. Therfore loke what proportion 7 beyng this totall, doth beare to 5, and to 2, suche proportion shall any totall in this question beare to the pasture ground and the eared grounde. Schol. This serueth wonderous aptely. Therfore to proue it I demaunde this by the former suppositiō: If a man haue 300 acres, how muche shall be leane in pasture, and how muche shall be tourne to tyllage? You say that as 7 is to 5, so shall 300 bee to the akers of pasture, and as 7 is to 2, so is 300 to the akers of tyllage, whereof for bothe I haue set examples here folowing, whereby appereth that of pasture there shall bee 214 2/7 akers, and of tyllage, 85 5/7, whyche bothe summes added togyther doo make 300. M.

7 — 5 — 300 — 214 2/7

7 — 2 — 300 — 85 5/7

other question.

Nowe take an other example: A man hath iii. syluer cuppes with one couer, the couer wayeth 18 vnces, the seconde cuppe weyeth euen halfe the weyghte of the fyrste and the thyrde. Nowe if the couer be put to the fyrst cuppe, they waye iust as much as all the iii. cuppes doo waye, and the couer bee toyned with the seconde cuppe, they waye as muche as the seconde twyse and the thyrde, and yf the couer bee put to the thyrde cuppe, they wyll waye twyse as muche as the fyrste and seconde cuppe. Nowe trye you what was the

iuste

ſyſte weight of euery cuppe. 5. I dos ſet the weyght of the fyrſte cup to be 9 vnces, euen in as much as thole thoodo wait the weight of the .iij. cuppes. I ſee that the .iij. cuppes muſt way 27 vnces, for ſo much is 18 and 9. Alſo becauſe the fyrſte and the ſeconde boge way double ſo muche as the ſecond, therfore is it the thyrde parte of that weyght that is 9, and then maie it folowe, that the thyrde cuppe alſo ſhuld way 9 vnces, but then the queſtion ſayth, that the couer beyng ioyned to the ſeconde cuppe, they way as much as the ſeconde tuyſe and the thyrde ones, that ſhuld be 27: and ſo it dooth. then beyng ioyned with the thyrde cuppe, they ſhuld waye as muche as the fyrſt and the ſecond tuyſe, that ſhulde be 36, and they way but 27, ſo is that errour 9 to littell. Then begynne I a gayne, and ſay that the firſt cup bothe waye 12 vnces, which I ioyne with the fyrſt cup, and they make 30 vnces, then ſeyng the ſeconds is $\frac{1}{3}$ of that weyght, it muſte nedes way 10 vnces, and the thyrde muſt waye 8 vnces, ſeyng the fyrſt and thyrd muſte way 20. Now put I the couer to the ſecond cup, and they waye 28 vnces, whiche ſhuld be euen ſo, thenne ioyne I the couer with the thyrde cuppe, and ſo ſhoulde it waye the fyrſte and the ſeconde tuyſe, that is 44 vnces, and they boge weye but 26, that is 18 to lyttelle, thole errourſe with theyr

positions I set down, and multiply its croste waite 9 by 12, whereof commeth 108. Also 9 by 18 & that yelderh 162. And in as much as the lignes be like,



I abate the lesser out of the greater, & there booth remayne 54. Then doo I also abate the lesser errour from the greater, and so remayneth 9, by whiche I divide 54, and the quotient is 6, whiche I take for the trewe weight of the fyrst cuppe, which beyng joyned with the couer muste waye as muche as the iij. cuppes, so doo they way but 24 vnces. Then seyng the second cup is the third part of that weight, for the other iij. cuppes (you say) must waie double his weight, the weight of the seconde cuppe is 8 vnces, and so the weight of the thyrd must be 16 vnces. Now put the couer to the second cuppe, and it will make 26 vnces, & must be the weight of the second twise, and the third ones, that is twise 8 and ones 10, and so is it. Agayne put the couer to the thyrde cuppe of 16 vnces, and they must way as much as the first and the second twise, that is 28, and so is all agreable. M. Then answere this question. There is a cesterne with iij. cockes, conteinyng 72 barrels of water, And if the greatest cocke be opened, the water will auoyde cleane in vi. houres, at the seconde cocke it will alle 8 houres, at the thyrd cocke it will

Question of water.

auoyde

anoyde in no leſſe then 9 houres, and at the ſmalleſt it wyll require 12 howres. Now I demaunde in what ſpace will it anoyd, all the cockes being ſet open? Sco. Fyrſt I imagyn that it wyll anoyde in 2 houres. M. Then muſt there anoyd by the fyrſt cocke $\frac{1}{3}$ of the water, that is 24 barrels, and by the ſeconde cocke $\frac{1}{4}$, that is 18, and by the thyrde cocke $\frac{2}{5}$, that is 16 barrelles, and by the ſmalleſt cocke $\frac{1}{6}$ that is 12 galons, all whiche ſummes put togyther doo make 70, as by their addition it doth appere, but it ſhulde be 72, therfore the error is 2 to lewe. Sco. Then I begyn agayne by your fauor, bycauſe I thynk I vnderſtand the worke, and I put 3 houres for the due tyme, ſo ſhall there run out at the greateſt cocke $\frac{1}{3}$, that is 36 barrels, and at the ſecond hole $\frac{3}{4}$, that is 27, & at the thyrde cocke $\frac{1}{3}$, that is 24, and at the ſmalleſt hole $\frac{2}{5}$, that is 18 barrels, whiche all togither do make 105, and ſhuld bee but 72, ſo is it to much by 33. Therfore do I ſet the errors in order of the figure with their poſitions, and worke by multiplicatiō in croſſe, ſayenge 2 tymes 33 is 66, and 3 tymes 33 maketh 66, and bycauſe the figures are vnlyke, I muſt adde thoſe .ii. totals togither, whych make 72, and I adde the .ii. errours, and they make 35, by whyche I di-

24	
18	
16	
12	
70	

2 3
2 33

uide 72, and the quotient ryseth $2\frac{2}{3}$, wher by I see, that all the cockes beyng set open, the water wyll auoyde in 2 houres, and $\frac{2}{3}$ of an houre. M. This exercise maketh you to growe experte in the rule, therfore I wyll inure you yet somwhat more with a queſtion of .ij. There were .ij. men that had bene partners, and had in accompt betwen them 300 buckers, wherof the one ſhuld haue for his part 180, and the other 120, but in the partyng of theym, they fell at variance, ſo that eche of them catched as manny as he coulde, yet afterwarde beyng reconciled they agreyd, that he which had gotten moſt part of them, ſhuld lay downe $\frac{1}{3}$ of them agayn, and he that had gotten leaſt, ſhuld lay wune $\frac{1}{3}$ of thoſe whyche he had taken, and then partynge them into .ij. equall partes, eche man to haue halfe thereof, and ſo had they their iuſt portions as they ought. Nowe I demaunde of you what eche of them hadde gotten by the ſkambyng. Scho. I ſuppoſe, he that had leaſt, gat 108 ducates, then the other had 192, wherfore in layeng downe agayne of the 192, there was put downe $\frac{3}{4}$, that is 144, and ſo had he left but 48. Alſo of the 108 there was layde downe 36, that is $\frac{1}{3}$, and ſo he had lefte 72. Then I put together 144, and 36, and it maketh 180, whyche I parte into two partes euen, and ſo commeth 90 to be gyuen for the one of them,

queſtion partners

whyche

which sum put to 72 maketh 162, & ioyned to 48, it maketh 138, & now I doubt how I shal go forward. M. You neede not to take but one of the, which you lyst, the greater or the smaller, for it cometh to one purpose, and so may you compare it, that you take to any of the other sumes, remembryng that you make the partion to the same in the second worke, as for exãple of the first part, if you compare 138 vnto the lesser sume due, & is 120, so is it 18 to much, & if you cõpare it vnto the greater sum, thã is it 42 to litle. Again if you cõpare 162 to the greater sum, the errour will be 18, as it was in the other, but it wyll haue a contrary signe, & if you cõpare it vnto the lesser summe, it will be 42 to much, so that the errour both wais is either 18 or 42, & as for the signes, it litle forceth, for in the is nothyng cõsidered here but likenes & vnlykenes, which in this case doth neither further nor hynder. But now go on with the worke. S. If it be so, than am I out of my greatest doubte. Then I ioyne that 90 (which I found as the halfe of the latter partitiõ) vnto 48, which is left with the one man, & so hath he 138, which (I may say) is 18 to manye, for the least should be but 120, that errour do I note, and then make a newe position, supposynge the one man to haue 204, and the other to haue 96, wherefore of the 204 there muste be layde downe 153, and so remaynyth with hym 51.

Note.

Also of the 96 there must be layde downe $\frac{1}{2}$, that is 32, and so resteth with that man 64. Now of the 153 and 32 I make one summe as 185, whiche I muste diuide into 2 equall partes, and so ethe manne shall haue $92\frac{1}{2}$, whervnto yf I adde their former portions reserued, then the one shall haue $156\frac{1}{2}$, and the other hath $143\frac{1}{2}$. Wherfore I take the lesser summe nowe agayne as I dyd before, that is $143\frac{1}{2}$, and fynde that he hath to many by $23\frac{1}{2}$, for he shuld haue but 120, so haue I for my .ii. positions .ii. errours, whiche I sette downe as here may be sene, ethe errour vnder his position, and thā by the rule I doo multiply in crosse wayes 108 by $23\frac{1}{2}$, and there riseth 2538 which I note, then agayn I multiply 96 by 18, and therof amounteth 1728. Now bycause the signes are bothe lyke, that is bothe to many, I must worke by subtraction, & so abatynge 1728, out of 2538, there wyll rest for the diuidende 810, then for the diuisour I subtract 18 out of $23\frac{1}{2}$, and there remayneth $5\frac{1}{2}$, by whiche I diuide 810, and the quotient wyll be $147\frac{3}{11}$, whiche is the iuste portion of hym that had the least sum. And if I do subtract it out of 300 beyng the totall summe, then wyll there remayne $152\frac{8}{11}$ as the portion that the other dyd gette.

108 96

18+ $23\frac{1}{2}$+

Mat. For the proofe of this worke, you may chose

thole whether you wyll examyn thole numbres accordyng to the forme of the queſtion, or els worke by other .ii. poſitions, for to fynde the ſeconde noumbre, and if thole poſitions bryng the ſame numbres that dyd amounte by the fyrſt .ii. poſitiõs, then dothe eche worke confirme other. S. By your pacience I will proue bothe waies, not only to ſee their agremente, but alſo to accuſtome my mynde to thoſe wookes: for I perceaue it is exerciſe, that muſt be the chiefe engrauer of theſe rules in my memorie. M. You conſyder it well, then go to. Sco. Fyrſt I will by. ii. other poſitions trie to fynde the portion of hym, whi che had moſte. M. Although you may doo it with any poſitions, yet to ſee the agrement of your worke the better, take the ſame poſitions that you dyd before, comparyng them nowe to the greater, as you dyd before vnto the leſſer. Sco. Then I ſuppoſe that he that had moſte, had 192, ſo had the other 198. Nowe if I take ¾ of 192, that wyll be 144 and there wyll reſt to that man but 48. And from the ſeconde whiche had 108, if I take ⅓, that is 36, there wyll remayn to hym 72. then toynting 144 with 36, it will make 180. the halfe whereof beyng 90, if I adde to eche of thoſe .ii. mens portions remayning with them, the one ſhall haue 138, and the other 162, of whiche .ii. I take the greater (that is 162) and ſee it to be 18 to fewe, for it ſhuld be

180, that errour I note vnder his position. Then for the seconde position, I take (as I dyd before) 204 for the one, and so resteth 96 for the other, then take I $\frac{1}{3}$ of 204, and it wyll be 153, and there resteth to hym 51, also of the 96 I take $\frac{1}{3}$ that is 32, & there remayneth to hym 64. Now put I that 32 vnto 153, and it yeldeth 185, whyche beyng parted in equall halues maketh 92 $\frac{1}{2}$, to bee added to eche mans remaynder, and so the one hath 143 $\frac{1}{2}$, and the other 156 $\frac{1}{2}$. wherfore I take the greatest summe, and it is 23 $\frac{1}{2}$ to lyttell, that doo I note also, and sette bothe these errours vnder theyr positions, as in this example dooth appeare. And then multiplyeng 192 by 23 $\frac{1}{2}$, there doothe aryse 4512. Agayne I multiply 204 by 18, and it maketh 3672, whyche I do subtract out of 4512, bicause the signes bee lyke, and there resteth 840 for the diuidend, then subtractyng 18 out of 23 $\frac{1}{2}$, there will remayn 5 $\frac{1}{2}$, which I must take for the diuisor. And so diuidyng 840 by 5 $\frac{1}{2}$, the quotient will be 152 $\frac{8}{11}$, wherby I haue founde an agreable sum to that whyche I found by the former positions, for him that had most, whyche if I do subtract out of 300, (that is the totall) there wyll rebe 147 $\frac{3}{11}$, whyche was the portion of hym that had the leaste

192		204
18 —		23 $\frac{1}{2}$ —

parte

parte. Mai. So by yuers positions you see that one wothe tonfytme the wozke of the other. Nowe examyne thole. ii. numbzes by the tozme of the question, and so shall you proue youre wozke good also. Scho. If that he whiche gat moste had 152 $\frac{8}{11}$, then must he lay dowbne $\frac{3}{4}$ of his summe, that is 114 $\frac{6}{11}$, and so shall remayn with hym but only 38 $\frac{2}{11}$. The other whiche had leaft, that is 147 $\frac{3}{11}$, must put downe of his summe $\frac{1}{3}$, that is 49 $\frac{1}{11}$, and so dothe there remayne with hym yet 98 $\frac{2}{11}$. Then doo I adde togyther 114 $\frac{6}{11}$ and 49 $\frac{1}{11}$, and it wyl make 163 $\frac{7}{11}$, whiche I must part into two equal partes, and that wyll bee 81 $\frac{9}{11}$, to be geuen to eche of them. so puttynge 81 $\frac{9}{11}$, vnto 38 $\frac{2}{11}$, there dothe amount 120 iuste, whiche is the true porzion of hym that shuld haue the lesser summe, and addyng 81 $\frac{9}{11}$, to 98 $\frac{2}{11}$, the totall will be 180, the true porzion of the other. And so is the wozke by this prose also feied to bee good. And this I marke by the waie, that in their scambling, he gat moste, (as it chaceth often) that ought to haue had leaft by iuft partitiõ. M. Let your studye be to learn truely & iuft art of proportiõ, & to distribute & part accozdyng therunto, as often as occasion shalbe ministred. And here wouldo I make an ende of this rule, saue that I remembze one pleasaunt question whiche I can not ouerpasse, whyche I wyll declare

Z.v. some

somewhat largely, because you shall as well vnderstande some reason in the pleasant inuention, as apt proceedyng in the witty worke kyng therof. Hiero kyng of the Syracusians in Sicilia had caused to be made a croune of golde of a wonderfull weyght, to be offered for his good successe in warres, in makynge whereof the goldsmyth fraudulently toke out a certayn portion of gold, and put in syluer for it, so that there was nothynge abated of the full weyght, although there was muche of the value diminyshed, whiche thynge at length beynge vttered (as no euyll can always lye hydde) the kyng was sore moued, and beyng desyrous to trye the truthe without breakynge of the crowne, proponed the doubte to Archimedes, vnto whose wytte no thyng semed vnpossible, whyche although presently he couldo not answere vnto, yet he had good hope, to diuise some policy for that inuention. And so musynge thereon, as he chaunced to enter into a bayne ful of water, to washe hym, he obserued that as his body entred into the bayne, the water dyd runne ouer the tubbe, wherby his ready witte of suche small effectes conuecturyng greater workes, conceaued by and by a reason of solution to the kynges question, and therfore reioycyng exceedyngly more then yf he had gotten the croune in deede, forgat that he was naked, and so ranne home, crying as he ranne

εὕρηκα, εὕρηκα, I haue found, I haue found. And therupon caused ii. massy peces one of golde, and an other of syluer to be prepared, of the same weight that the said crown was of, and considering that golde is heuyer of nature then syluer, and therfore gold of like weight with syluer must needes occupy lesse roume, by reason it is more compacte and sound in substance, he was assured that puttyng the masse of golde in to a vessell brym full of water, there wolde not so muche water runne ouer, as when he shulde put in the syluer masse of the lyke weyght. Wherfore he tryed both, and noted not only the quantities of the water at eche tyme, but also the difference or excesse of the one aboue the other, whereby he learned what proportion in quantitie is betwene golde and syluer of equall weyght. And then puttyng the croune it selfe into the vessell of water brymfull as before) marked howe muche water dyd runne out then. And comparyng it with the water that ranne out when the golde was put in, noted howe muche it dyd excede that, lykewaies comparyng it to the water that ranne oute for the syluer, marked howe muche it was lesse then that, and by those proportions founde out the iuste quantitie of gold that was taken out of the crowne, and how muche syluer was put in stede of it. But sayng Vitruuius whiche writeth this history

doth

boothe not declare the particular woorke of this triall, it shall bee no inconuenience to suppose an example for declaration sake, wherin although the true and full proportions be not expressed, yet the forme of triall shall be truely set foorthe. And for an example I suppose the weight of the croune to be 8 li. and so of eche of the other. ij. masses. And when the masse of golde was put into the water, I ymagine that there ranne out 2 pound of water, and when the masse of siluer was put in, I suppose there ranne oute 3 poũd $\frac{1}{2}$. Agayn when the croune was put in, there ran out 2 pound $\frac{1}{2}$. Now to knowe what quantitie of syluer was in the croune, woorke by the rule of false position, and imagyn that there was 2 pound of syluer, then must ther be 6 poũd of gold. Then say thus by the rule of proportion, If 8 pound of gold do expell 2 li. of water, what shall 6 li. expell? and it wil be 1 pounde $\frac{1}{2}$. Agayn for the siluer, if 8 pounde of syluer expell 3 pounde $\frac{1}{2}$ of water, what shal 2 poũd of syluer put out? it will be $\frac{7}{8}$. Nowe adde those two weightes of water together, and they wyll make two pounde $\frac{3}{8}$, and it should be by the supposition 2 li. $\frac{1}{2}$, so is it to muche by $\frac{1}{8}$. Scho. Now doo I vnderstande the woorke (as I think) therfore I praye you, let me woorke the rest of the question. And bicause this firste supposition dyd erre, I note that position and his er-

roure

roure. And take a newe position, estemynge the siluer to be but one pound, so must there bein golde 7 pound. Then say I, if 8 pound of gold yelde 2 pounde of water, what shall 7 pound yelde? and it will be 1 ponnd $\frac{3}{4}$. Again if 8 li. of siluer expell 3 li. $\frac{1}{2}$ of water, what shall 1 pound expell? and it will be $\frac{7}{16}$. Nowe must I adde thole 2 summes togither, and they make 2 li. $\frac{3}{16}$, and they shuld make 2 li. $\frac{1}{4}$, so is it to lytle by $\frac{1}{16}$. Therfore I set the positions with their errours in ouer, as here folowech. And then I multeplye in crollewaies 2 by $\frac{1}{16}$, & it maketh $\frac{1}{8}$. like wats 1 multiplied by $\frac{1}{8}$, maketh $\frac{1}{8}$. And bicause the lignes be vnlike, I must adde thole 2 summes, which make $\frac{1}{8}$, that is the diuidend. Agayne I must adde $\frac{1}{8}$ to $\frac{1}{16}$, and it will be $\frac{3}{16}$, that is the diuisour. Now I shall diuide $\frac{1}{8}$ by $\frac{3}{16}$, and the quotient will be $\frac{16}{24}$, that is $\frac{2}{3}$, wherby I knowe that there was put 1 pound and $\frac{2}{3}$ of syluer into the croune, and so muche golde taken oute for it. Mayster. Proue it nowe by examination, accordynge to the question. Scolar. If there were 1 pound $\frac{2}{3}$ of syluer, then was there of golde 6 pounde $\frac{2}{3}$. Nowe saye I by the rule of proportion, yf eyghte pounde of golde expelle two pounde of water, what shall 6 pounde $\frac{2}{3}$ expelle? It will bee one pounde $\frac{2}{3}$ Agayne

Agayn yf 8 li of syluer expell 3 li $\frac{1}{2}$ of water, what shal 1 $\frac{1}{2}$ expell? It will be $\frac{3}{1\frac{1}{2}}$. Now must I adde together 1 li $\frac{2}{3}$ and $\frac{3}{1\frac{1}{2}}$, and they wyll make 2 li $\frac{3}{1\frac{1}{2}}$, that is 2 li $\frac{3}{4}$, accordyng to the suppolition of the question, whereby I perceaue the worke to be well don. And as I can not but muche reioyce of this excellent inuention, so my desyre is kyndled vehemently to be perfectly instructed in euery parte thereof, and namely in this poynt, whether the proportion betweene water and golde be suche, that for 8 li of golde put into a vessell full of water, there shall runne out two pounde of water. And for as muche syluer, whether 3 li. $\frac{1}{2}$ of water wold anoybe? M. I perceiue your meanyng, and coniecture your imagination to be thus, that if you knewe the exact proportion betweene gold syluer and water both in their waight and in their quantities, then coulde you easyly fynde out the mixtures of them, whiche thyng I haue referred for an other worke that intreateth suche matters specially. And at this tyme you muste consyder, that you learne Arithmetike, whiche intreateth of the maner to solue doubtefull Questions touchynge numbre, without regarde what matter is signified by that numbre, els were it necessary in Arithmetike to teache all artes, seeyng in it may be moued questions of all artes. But seeyng you are so

desy-

delyrous to knowe this thyng, I wyll tell it you in suche a lorte, that you shall practise yourselfe in fyndyng it. And propound it in forme of a question, Gold beareth greater proportion to water then syluer dothe, and their two proportions be in proportion together, as $\frac{4}{2}\frac{8}{5}$. But to healpe you somwhat in this riddle, you shall note that the proportio of quicke syluer vnto water is the iust myddle numbre proportionall in progression geometricall, betwene the proportions of golde and syluer vnto water. And his proportion is, as $\frac{200}{121}$. Now yf you will know the iuste numbres of these 3 proportions, then muste you finde out 3 numbres in progression geometricall, whereof the myddlemoste must bee $\frac{200}{121}$, and the fyrst must be vnto the laste, as 25 to 48. And thus I wyl leaue you to fynd those numbres, when you be at leisure. &c. Yet fyr I thanke you hartily for this much. for nowe I see the possibilitie to fynde them out. Howbeit bycause this question semeth strange, it it myght pleale you to instruct me somewhat in the order of woorkyng for it, I shuld the more easyly fynde the freewe woorkynge. Mast. You delyre to muche eale, yf you wyll study for nothyng: therfore to occasion you to study the better, I wyl leaue this doubt wholly to your owne practise. But as touchynge the generalitie of the rule, Archimedes vsed not to take two masses of golde and

I shuld fynde of the proportion of gold, syluer and quicke syluer vnto water.

and syluer equall in weyght with the crowne for the proporcion wyght as well be founde in any other weight, yea although the masse of golde were of one weyght, and the masse of syluer of an other. As for example, If the crowne were of 8 pound weight, as I did suppose, and I haue not so muche other fine golde, but onely 1 pound, and tryeng that by water, and fyndyng that it dothe expelle but ½ of an vnce of water, yet than by it maye I inferre, that 8 pound of golde wold expell 6 vnces of water. And lykewaies of the syluer wherof if I had but 1 pound, and fynd that it doth expel iij. vnces of water, then might I affyrme that 8 pound wolde expell 12 vnces, that is one pounde weight. And so is it as good as yf the 3 masses were all of one weyght. And thus for this tyme, I wyll make an ende of this other parte of Arithmetike. S. Although I can not sufficiently thãk you for this, yet your promise made me to loke for the arte of Extraction of rootes wherof hytherto I haue learned nothyng. Maister, I wyll not breake my promyse, but intende (god wyllyng) to performe it within these thre or foure moneths, yf I perceyue this my payns to be well taken in the meanseason. And you shall not repent the taryeng for it, for it shalbe increased by the taryeng.

And in the meane tyme you shall take this addition not for the leaste parte of Arithmetike

mente, whiche I promysed, but for an augmentation of the fyrst parte, vnto whiche I wold haue annexed the Extraction of rootes square and cubike, namely for exemples of the statute of assise of woodde, but that in the seconde part I must write of diuers other rootes, and therfor thought it best to reserue those rootes also with their exemples to the same seconde part. Scholar. Syr although I can not recompence youre goodnesse, yet I shall alwayes doo myne indeuour to accasion you not to repent youre benefite on me thus imploied. Maister. That recompence is sufficient for your parte.

The fyrste letter signifieth the quayre, the seconde the page, a, the fyrst syde, b, the seconde.

A.v.a. line 18, in the third exaple set 2 for 11
D. ii. a. in the exaumple, set 9, for 6, in the seconde rowe.
D. iiij. a, in the lowest examples hewes are pl' mured with ij, and likewise the cees drawen to and vnder the kues out of order.
In the same page, These 4. summes, Rede These 8 summes.
D. viij. a. for all alyke, rede all like. Item on the same side the last line, for hall you double, reade, shall you double.
E. iii. b, in the third example on the left syde lacketh a strike or line aboue 12.
E. v. b. line 26 put out one, in.
E. vi. b. in the lynes that come benethe the summes togither must be brought lower from the vpper sum to the seconde summe, and the other part ending euen with 64.
Idi. line 12 read, lerned before comon of ce
F. i. b. line 11, reade, to doo it therfore. &c.
F. ii. b. line 10, which cometh of the iii. parcels. enclose with a parenthesis.
F. iiii. a, line 3 for, alone, reade and all one.
F. v. a. set the first exaumple on the left syde.
F. viii. a. line 9, reade (as the reason wylleth me. On the same side in the lowest exam

ple

ple, bryng thes two nombre placed vnder 6) backe into the seconde place right vnder 4 and 3.

	364
F. viii. b. mende the example thus,	29
	1336
	184
	28

B. i. b, the first example mende thus, and set it on the lefte syde of that page.	1365 236 1330 686 15	Item on the same syde, the fourth example, ye

must sette this nombre 398, 8 vnder 1, and 9 vnder 1, &c.

Also on the same side the. v. example on the right side, the third sum beneth the line, is 151 whiche shuld be 115.

Item on the same side the next example folowyng toward the left side mende the playng of the figures.

B. ii. a, the figures of the example in the margent on the lefte side all vnder the line are falsely placed, set them therfore ryght thus.	1365 326 1330 686 115
B. ii. b. in the fourthe example make the multiplier 365.	398 110
B. iii. a, in the firste example the lowest sum 252 shuld be set vnder	771.
B. iii. b first exaple lacketh a line vnder	365

B.b.a, line 5, for 3 vnder the barre, reade 8 vnder the barre.

Also in the lowest example on the right syde where the vppermoste lyne is made vnder 18, sette it vnder 23 aboue 18.

B. viii.a.lin.10, for thole 363, rede those 365 ther the secõd exãple hath his diuisor 52 vnder 36, set it vnder 65. there in the lowest example 28 vnder 65 set backe vnder 36.

B. viii.b, in the nether example the viii. figures x 8 shulde be sette ouer x 6 viii, that is x ouer x, and 8 ouer 6.

D. i. a, in the lowest example, the lowest figure 2 put vnder the second 8, and the vpper 2 and 5 in the mydde rowe with 8 vnder hym, leaue vncancelde.

D. i.b, in the seconde exam- 136289
ple place the figures thus. 452

D. viii.a, the second example lacketh the figure of 1 on the right side of the crosse. also 21 line. rede, and they 2, that &c.

D. b. b, in the lower example in the mydde numbre, 765, lacketh a 6.

D, vi.b. in the first example 4331010 is set 3 for 5, mende it thus. 4531010.

I. viii.a, there lacketh the figure of 1 in the lowest example on the right hande.

I. vi. a, the diuisour in the fyrste place is made 74 for 64. line 5. Ibidem line 30, reade for numreb, numbre.

K. 4. line 13, rede, and so of other in like rate

R. v. b. lin. 12, for 196 pond. Rede, 256 pond.
R. vi. a, 25, line, 93 gallõs, rede 63 gallõs.
L. i. a, line 24, 2, 8 32, mend it thus, 2, 8, 32.
L. ii. a. line 8, for, Rede, for the second 2 ob.
L. iiii. b. line 4. rede amonteth 128.
L. 8. a, mend the 20 — 360
b. example thus, 622980 — 7464960
M. vii. b. line 14. for 28796, rede, 287965.
P. vi. a, last line mende the exãple thus: $\frac{1}{12}$.
P. vi. b. 10 line, for the numeration, rede their numeration.
P. vii. a, line 4, mende the fractiõ thus, $\frac{2}{12}$
Ibi. 5. line, mende the iii. fractions thus, $\frac{3}{12}$, $\frac{4}{12}$, $\frac{6}{12}$.
P. 8. b, the 24 line, for $\frac{4}{5}$ $\frac{2}{5}$, set $\frac{4}{5}$ $\frac{2}{5}$,
Ibid. the last line, for $\frac{4}{5}$ put $\frac{4}{5}$.
M. i. a. the fourth line, for $\frac{4}{5}$ $\frac{2}{5}$ make $\frac{4}{5}$ $\frac{2}{5}$.
Ibi. 13 lin. mẽd ye 2 first fractiõs thus, $\frac{4}{5}$ $\frac{2}{5}$.
Ibi. 17. line, for fraction, rede fractions.
M. iii. a. the 16 line, for $\frac{5}{12}$, make $\frac{5}{12}$.
M. iiii. b, ye 8 line, rede, first I must multiply.
M. b. b. against the endes of the 9 and 10 line, for $\frac{2}{3}$ $\frac{3}{4}$ make $\frac{2}{3}$ $\frac{3}{4}$.
R. vi. a, lin. 5. for as $\frac{7}{100}$, rede as by $\frac{7}{100}$
R. vii. a, line 18, for, And if, make as if.
R. viii. a, line 12, for, 7 by 80, rede 7 by 20.
S. i. a, lin. 7, make $\frac{5}{12}$, in a line thus, $\frac{5}{12}$.
S. ii. b. line 1, for. 4 thacts rede 4, by whi che I must multiply ye other 4, that is er.
S. iii. a lin. 26, mende thus, $\frac{3}{400}$.
Ibid. line 27. the third fraction $\frac{1}{4}$, mende

a. iii, thus.

thus, $\frac{3}{40}$. Ibi. line 28 for $2\frac{3}{40}$ make $\frac{3}{40}$.
D. iii. b line 18 for $\frac{3}{2}\frac{4}{3}$, make $\frac{2}{3}\frac{4}{2}$.
D. viii. a. line 28 for, a pounde weigh, reade
pound weight. Ibi. b ii. 6 for $\frac{1}{20}$, make $\frac{1}{20}$
D. viii. a in the firſt columne of the table in
the 11 ſpare enclosed betweene 2 lynes, for
5 ß 5 ß, make 5 ß 6 ß.
T. iii. in the ſecond table of the weyghte of
the peny white loſe, for 24 ſpare, for 0 pens
$\frac{8}{25}$, make 2 pens, $\frac{8}{25}$.
T. vii. b, the lower exaumple, a,
mende thus, 240 ⟋ 8190
104 ⟋ 3549
Ibidem, the lower exaumple. c, mend thus,
240 ⟋ 8190
40 ⟋ 1365
T. ii. a, the fourth exaple, b, mende thus.
21 ⟋ 2970
3 ⟋ 424 $\frac{2}{3}$
T. ii. b. laſte line, reade, and 6 ouer of the
totall ſumme.
W 4 a, lin. 15, 9 6 4, diſtinct thē thus 9, 6, 4.
W 4 a, laſte lyne, Reade thus, $\frac{1}{2}$ by $\frac{1}{3}$, and
the quotient wyll be $\frac{3}{1}$.
W. viii. a, lins 13, for mixtion, reade mixture.
W. viii. a, in the lower exaumple, the ſyxt
ſumme 251, mend thus 250.

E. i. a in the fyrſt exaumple for	1	make 5
	y	1
	3	3
	&c.	&c.

E. i.